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ABSTRACT

This guide is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12. It is inclusive also of the objectives tested by the Stanford Achievement Tests and the Alabama Basic Competency Tests. One characteristic of the curriculum guide is that it clearly states what students should learn in each grade level. These objectives are called student outcomes and appear in the table of contents. The majority of the material, however, consists of classroom activities that lead students to the desired outcomes. The curriculum guide specifies what to teach as well as ways to teach it. Each section of the guide contains a specific table of contents, which lists what students should learn; activities; original sources; and student outcomes. Activities are preceded by a description of the area of mathematics covered, the specific student outcomes that are addressed in the activities, and notes for the teacher, which convey information important for effectively carrying out the activities. Topics include geometry concepts, points, lines, planes, angles, triangles, similar polygons, circles, geometric solids, locus, coordinate geometry, and transformations. (KR)

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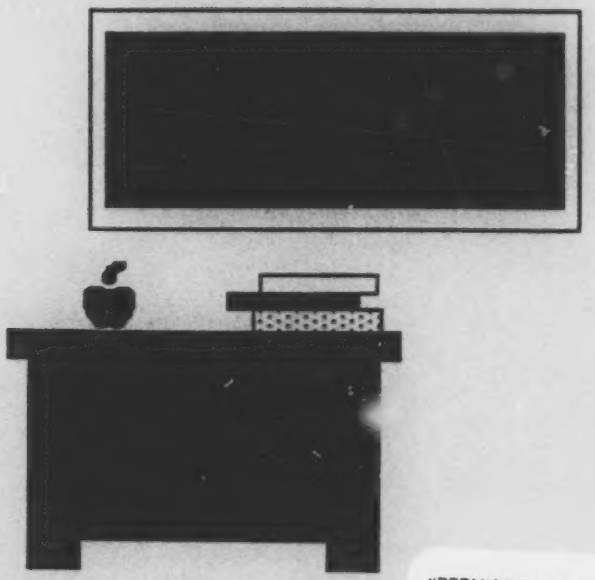
MATHEMATICS

CURRICULUM GUIDE UNIFIED GEOMETRY



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Bulletin 1987, No. 53

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State Superintendent of Education
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MATHEMATICS
CURRICULUM GUIDE
UNIFIED GEOMETRY

ALABAMA STATE DEPARTMENT OF EDUCATION

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FOREWORD

This Mathematics Curriculum Guide: Unified Geometry is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12 (Bulletin 1984, No. 16). It is inclusive also of the objectives tested by the Stanford Achievement Tests.

One characteristic of the curriculum guide is that it clearly states what students should learn in Unified Geometry. These are called STUDENT OUTCOMES and appear in the Table of Contents. The majority of the material, however, consists of classroom ACTIVITIES that lead students to the desired outcomes. The curriculum guide, then, specifies what to teach as well as ways to teach it.

The Mathematics Curriculum Guide: Unified Geometry is supplemental in nature. It will add to material found in textbooks and complement locally developed skills continuums, scope and sequences, and/or curriculum guides. In the past, we have been most successful when we clearly defined what students should learn, gave examples of how it can be taught, and then tested for mastery. This curriculum guide is another step in that direction.

Wayne Teague

Wayne Teague
State Superintendent of Education

INTRODUCTION

The Mathematics Curriculum Guide: Unified Geometry is one of a series of curriculum guides to be developed by the Alabama State Department of Education. Its production is part of a larger plan to provide LEAs with a curriculum guide in every subject area K-12, immediately following the Course of Study adoption in the same subject area. The Mathematics Curriculum Guide K-6 is packaged separately. Each of the other secondary mathematics guides (Grades 7 and 8, General Math, Consumer Math, Pre-Algebra, Algebra I, Algebra II, Trigonometry/Analysis, and Applied Vocational Math) will be packaged separately also and will be published shortly.

The Unified Geometry guide has three distinct sections. It begins with a TABLE OF CONTENTS which lists what students should learn (student outcomes) in Unified Geometry. After each outcome, a page number(s) directs the reader to classroom activities that address the specific outcome.

The second and largest section of the guide consists of classroom ACTIVITIES. Activities are preceded by a description of the area of geometry covered, the specific student outcomes that are addressed in the activities, and Notes for the Teacher that convey information important for effectively carrying out the activities.

The third section is referred to as the ORIGINAL SOURCES AND STUDENT OUTCOMES portion of the guide. It summarizes the relationship of student outcomes to the original sources from which the outcomes were taken. The original sources for the Mathematics Curriculum Guide: Unified Geometry outcomes were the Alabama Course of Study and the Stanford Achievement Test objectives. This section of the guide will serve as a reference for local school systems that may wish to trace the source of each outcome or to rate each outcome as being basic, standard, or advanced.

The use of the Mathematics Curriculum Guide: Unified Geometry is not mandatory. Instead, it is offered as material that will help implement the Mathematics Course of Study and prepare students for the Stanford Achievement Tests. The Mathematics Curriculum Guide: Unified Geometry can be used as it is in local school systems that currently have no guide, or it can serve as a starting point for developing a similar guide locally. In either case, the involvement of local educators in the development of curricula is valued and encouraged. It is hoped that this guide will bring about local expansion activities such as writing outcomes for "optional" mathematics courses; adding to the list of outcomes for a given subject; writing additional activities in specific areas of identified weakness; rating outcomes as basic, standard, or advanced; and developing uniform end-of-unit tests.

ACKNOWLEDGEMENTS

From its inception, the development of curriculum guides has been viewed as a cooperative effort between specialists in the Division of Student Instructional Services and selected classroom teachers from the public schools of Alabama. The Curriculum Development Section, under the leadership of Katherine Mitchell, has the responsibility for coordinating this project.

Work on the Mathematics Curriculum Guide: Unified Geometry started in January 1986, when Linda G. Harris was selected as Project Director and the identification of student outcomes began. The second phase of the project took place during July and August when five classroom teachers joined State Department of Education specialists in the writing of activities that teach the identified outcomes. For twelve days the following teachers offered their best ideas:

James Davidson - Jasper City Schools
Patricia Edwards - Etowah County Schools
Myrtle Johnson - Eufaula City Schools
Ann Lilly - Tuscaloosa City Schools
Wanda Motes - Enterprise City Schools

Over a period of eleven months, a team of State Department of Education specialists identified and reviewed student outcomes, wrote and rewrote activities, read activities for clarity, and proofed various drafts. Robin Lanier contributed the graphics for the guide. Others who were directly involved with the Mathematics Curriculum Guide: Unified Geometry were Dorothy DeMars, Robert Glass, John Howard, and Linda Pledger. Shirley Dorman was the primary typist for the document; Rosemary Johnson assisted her on the second draft. We appreciate the work of all these people and their co-workers, who made the adjustments and sacrifices necessary to carry out their own programs and, at the same time, made the Mathematics Curriculum Guide: Unified Geometry a division product.

UNIFIED GEOMETRY

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AREA: INTRODUCTION TO CONCEPTS OF GEOMETRY: ELEMENTS OF GEOMETRY

OUTCOMES:
1. Understand the term "Euclidean geometry" and know the fundamental proposition of Euclidean geometry.
2. Know that other geometries exist and be able to discuss some general properties of these non-Euclidean geometry.

NOTE FOR THE TEACHER

One of the easiest ways to present this material is a lecture.

ACTIVITY 1 - OUTCOMES 1 and 2

Properties of an object that relate to its size, shape or position in space are called geometric properties. Which of the following represent geometric properties?

1. The sky is blue.
2. John is 6 feet 2 inches tall.
3. John weighs 185 pounds.
4. A yield sign is made of metal.
5. A yield sign is shaped like a triangle.
6. The top of a table is flat.
7. The top of a table is wood.

Items 2, 5, and 6 represent geometric properties. These represent an object's position or shape in space. The material, color, weight, etc., do not describe the object's position in space and, therefore, do not represent geometric properties.

Some studies of geometry divide it into two specific areas: plane and solid. Plane geometry involves the study of "flat" figures which have only two dimensions. These could be drawn on a flat surface or piece of graph paper, like a Cartesian plane. Solid geometry involves the study of 3-dimensional figures. We live in a three dimensional world. Most geometry courses today combine plane and solid. The first mathematician to devise a model of this 3-dimensional

ACTIVITY 1 - OUTCOMES 1 and 2 (Cont'd)

physical space and to describe it accurately was a Greek mathematician named Euclid. This geometry is called Euclidean geometry. Later, mathematicians attempted to construct other physical models of space and other forms of geometry. Two of these are hyperbolic and elliptic geometry.

The fundamental proposition of Euclidean geometry is that through a point outside a line there is exactly one line parallel to the given line.

Hyperbolic geometry is credited to three mathematicians: Bolyai, Lobachevski, and Savss. Its fundamental proposition is that through a point outside a line, there is more than one line parallel to the given line.

Elliptic geometry was developed by Riemann. Its foundation is that through a point outside a line there is no line parallel to the given line.

ACTIVITY 2 - OUTCOMES 1 and 2

Place books on geometry on reserve in the library. Have students read and write synopsis cards (5" x 8") to turn in on a weekly basis for the first grading period.

A recommended bibliography follows:

1. Mathematics Teacher (those issues that pertain to geometry.)
2. Numbers at Work, Hyman Alterman
3. Romping Through Mathematics, Raymond W. Anderson
4. Men of Mathematics, E. T. Bell
5. Mathematics Illustrated Dictionary, Jeanne Bendick
6. Non-Euclidean Gometry, Roberto Borola
7. Lapses in Mathematical Reasoning, V. M. Bradis
8. Introduction to Geometry, H.S.M. Coxeter
9. Euclid and Geometry, Estelle A. DeLacy
10. String, Straightedge, and Shadow: The Story of Geometry, Julia E. Diggins
11. Projection and Euclidean Geometry, W. T. Fishback
12. The Geometry of Art and Life, Matila Ghyka
13. The Dot and the Line, Norton Juster
14. The Education of T.C. Mits, L. Lieber
15. Geometry of Four Dimensions, Honey Manning

ACTIVITY 2 - OUTCOMES 1 and 2 (Cont'd)

16. Introductory Non-Euclidean Geometry, Henry P. Manning
17. Mathematical Snapshots, H. Steinhaus
18. Flatlands: A Romance of Many Dimensions, Edwin A. Abbott

NOTE: These are by no means all the books that are available or appropriate. Check your library for these and other books on geometry.

AREA: INTRODUCTION TO CONCEPTS OF GEOMETRY

OUTCOMES: 3. List, discuss and represent the undefined terms of Euclidean geometry.
4. Recognize the definitions of all defined terms of Euclidean geometry.

NOTES FOR THE TEACHER

Steps to Success

This is the first course in which the student can fully understand the basic structure of any mathematics course. Explain to the class that any math course is organized in the following way:

I. Undefined Terms

Those words and expressions that we use to talk about the subject area but for which we lack sufficient information to give a formal definition. In geometry they are: point, line, plane, and space.

II. Defined Terms

Those words and expressions for which sufficient background has been established to enable the formation of a precise definition. An example would be definition of polygons: triangle, quadrilateral, pentagon.

III. Postulates

Geometric (or other subject area) properties that are assumed to be true but never proven.

NOTES FOR THE TEACHER (Cont'd)

IV. Theorems

Properties that are proven to be true by use of undefined terms, defined terms, postulates, and other theorems.

V. COROLLARIES

A corollary is a special case of a theorem.

ACTIVITY 1 - OUTCOME 3

A roll of plastic wrap and some bright yarn are necessary. The students use the plastic wrap to represent planes, the yarn to represent lines, and their fingers or pencil tips to represent points. Use these "tools" to demonstrate how infinite numbers of planes, points, and lines exist. How, and in what manner, may things intersect and what "figures" are produced at their intersection. Concepts of perpendicularity, skewism, and parallelism may also be demonstrated using these tools.

ACTIVITY 2 - OUTCOMES 3 and 4

Ask the students to bring in pictures of examples of geometry demonstrated in the every-day world. They will find examples in nature, science, construction, architecture, art, transportation, household items, among others. Make a collage on the bulletin board. Have the students distinguish between the defined and undefined terms used in describing these examples.

ACTIVITY 3 - OUTCOME 4

As new terms are introduced, have the students include them in a notebook and write a formal definition of each. Periodically, utilize drill activities to review these definitions. Include a section on each test that requires a knowledge of definitions.

AREA: INTRODUCTION TO CONCEPTS OF GEOMETRY

OUTCOME: 5. Apply the principles of symbolic logic to the concept of proof.

NOTES FOR THE TEACHER

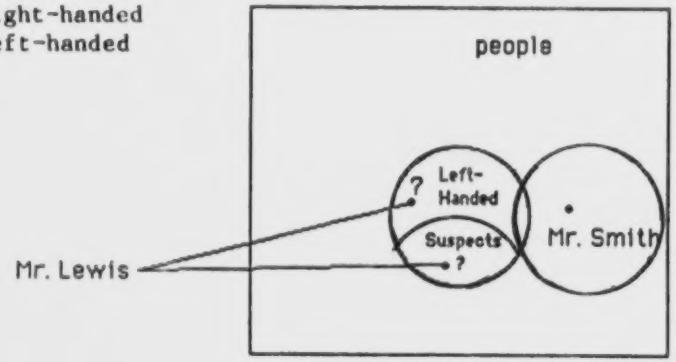
Venn diagrams are excellent for explaining set theory (subset, intersection, and union of sets) as well as deductive logic and reasoning. The following activities start with very simple diagrams and move to more difficult geometric proofs.

Steps to Success

The student should have the ability to transfer thoughts or concepts to diagrams or pictures.

Example: In a certain homicide case, the perpetrator must be left handed.

Mr. Smith is right-handed
Mr. Lewis is left-handed



The diagram shows that Mr. Smith could not have committed the murder, and Mr. Lewis may or may not be the murderer. The following conditional conclusion can be drawn:

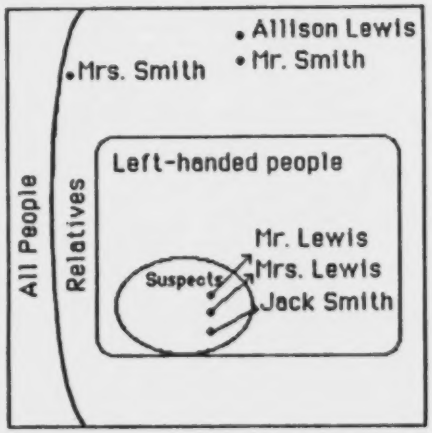
If a person is the murderer, then he is left-handed.

More evidence is introduced. The murderer was a relative of the deceased. The relatives are:

Mr. Lewis	Mrs. Lewis
Allison Lewis	Mrs. Smith
Mr. Smith	Jack Smith

In order to narrow down the list of suspects, determine who is left-handed and who is right-handed and insert this information into the diagram.

Mr. Lewis - left-handed
Mrs. Lewis - left-handed
Allison Lewis - right-handed
Mrs. Smith - right-handed
Mr. Smith - right-handed
Jack Smith - left-handed



The Suspects are now: Mr. and Mrs. Lewis and Jack Smith.

Have the class create more evidence which would narrow down the number of suspects even more. Some types of evidence might be:

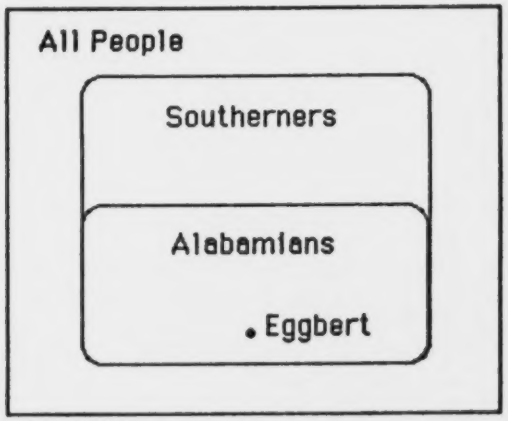
1. The murderer is male (or female)
2. The murderer must have been at the scene of the crime between 6 and 7 a.m.
3. The murderer must weigh less than (or more than) 150 pounds, etc.

ACTIVITY 1 - OUTCOME 5

Instructions: Have the students (1) express the following in If-Then form, (2) draw a Venn diagram of statements, (Point out that the If words are always a subset of the Then statement), (3) draw appropriate conclusions:

All Alabamians live in the South.
Eggbert lives in Birmingham, Alabama.

Solution: If a person is an Alabamian, then he or she lives in the South.



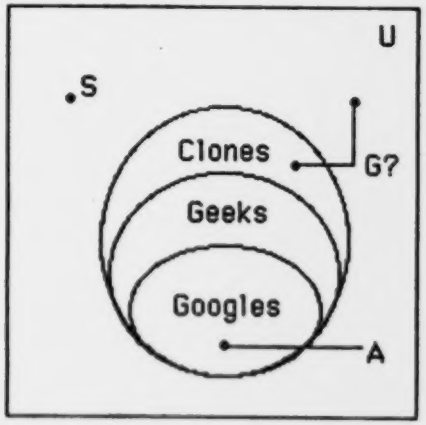
Conclusion: Eggbert lives in the South.

ACTIVITY 2 - OUTCOME 5

Have the students represent the following statements with a Venn diagram and draw appropriate conclusions:

If you are a google, then you are a geek.
All geeks are clones.
S is not a clone.
A is a google.
G is not a geek.

Solution:



Conclusions: S is not a clone, geek or google.
G is neither a geek nor a google, but could be a clone or none of the classifications.
A is a google, therefore, a geek, a clone, and a member of the universal set.

AREA: INTRODUCTION TO CONCEPTS OF GEOMETRY

OUTCOME: 6. Distinguish between inductive and deductive reasoning.

NOTES FOR THE TEACHER

Prerequisite for Students

Distinguish between general and specific statements.

Steps to Success

Emphasize that deductive reasoning is sometimes described as reasoning from the general to the specific, and that inductive reasoning can be described as reasoning from the specific to the general.

ACTIVITY 1 - OUTCOME 6

Explain the following examples which illustrate deductive and inductive reasoning. Lead students in a discussion where they describe other situations that use deductive and inductive reasoning.

Example 1: Suppose the following two statements are true.

1. Every beauty contestant is pretty.
2. Jane is a beauty contestant.

It can then be concluded that:

3. Jane is pretty.

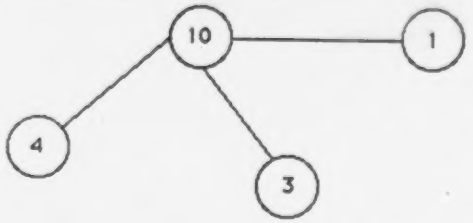
In this example some statements have been accepted as true, and reasoning proceeds from those statements to a conclusion. This is an illustration of deductive reasoning.

Example 2: Mark noticed that Mrs. Sims had been assigned to teach geometry for the past three years. He then stated: "Mrs. Sims will teach geometry this year."

This decision may or may not be correct, but it is based upon inductive reasoning. Induction is a process where individual cases are observed and a general principle is made based on the observations.

ACTIVITY 1 - OUTCOME 7 (Cont'd)

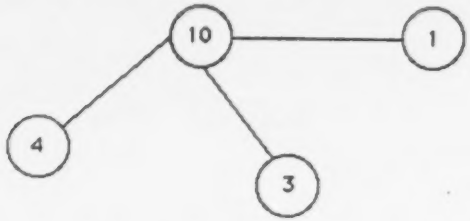
Before beginning, you may want to show students a sample method of drawing a map.



Teacher's Instructions	Possible student responses
You are on the bank of Deep Creek at stone 6. This stone is connected to stone 11, 9, and 7. To which stone do you want to move?	11
Stone 11 leads only into deep water. You must return to stone 6.	
You are on stone 6; it is connected to stone 11, 9, and 7. To which stone do you want to move?	7
Stone 7 is connected to stones 2, 3, 4, and 5. To which stone do you want to jump?	5
Stone 5 leads only to deep water. You must go back to 7.	
Stone 7 leads to stone 2, 3, 4, and 5. To which stone do you want to move?	2
Stone 2 leads only to stone 1 or back to stone 7. To which stone do you want to move?	1
Stone 1 leads only to deep water. You must go back to 2, which leads only to 1 or 7. To which stone do you want to move?	7

ACTIVITY 1 - OUTCOME 7 (Cont'd)

Before beginning, you may want to show students a sample method of drawing a map.



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Stone 2 leads only to stone 1 or back to stone 7. To which stone do you want to move?	1
Stone 1 leads only to deep water. You must go back to 2, which leads only to 1 or 7. To which stone do you want to move?	7

ACTIVITY 1 - OUTCOME 7 (Cont'd)

Teacher's Instructions	Possible student responses
Stone 7 leads to stones 2, 3, 4, and 5. To which stone do you want to jump?	3
Stone 3 leads to stones 8 or 10. To which stone do you want to move?	10

You Made It!!! You are on the opposite bank of the creek!

The steps may be listed, along with reasons, as in a formal proof.

	Statements		Reasons
1.	6	11	
2.	11	6	11 was a dead end.
3.	6	7	
4.	7	5	
5.	5	7	5 was a dead end.
6.	7	2	
7.	2	1	
8.	1	2	1 was a dead end.
9.	2	7	2 led only to a dead end.
10.	7	3	
11.	3	10	

ACTIVITY 1 - OUTCOME 7 (Cont'd)

The more complex the map is the longer the game.

Relate the game to geometric proofs. Point out that the steps leading to dead ends would be omitted in a formal geometric proof and only the shortest path would be taken.

The game is easy to relate to proofs in the following ways:

1. The "given" is the material stated at the beginning.
2. The conclusion or "prove" statement is the opposite shore or bank.
3. "Reasons" in geometry are definitions, postulates, theorems, or corollaries which lead (as the stones across the creek) to the "prove" statement.
4. Students may "jump to the wrong stone" in writing a geometric proof. Sometimes the elimination of steps may be necessary to derive the best proof.

Later theorems, corollaries, and postulates may be substituted in geometry for the numbers on the stepping stones.

ACTIVITY 2 - OUTCOME 7

Another form of proof that is frequently used in everyday reasoning as well as in the study of mathematics is called indirect proof. Most indirect proofs are written in paragraph form and contain at least three steps:

1. the original (contradictory) supposition;
2. the reasoning from this to the actual contradicition; and
3. the conclusion.

The proof begins by assuming the statement to be proved is <u>not</u> true and then continues from this until the contradiction of a previously known fact appears.

Sally's name appeared on the all A honor roll the first six weeks. The conclusion is that Sally made no grade lower than an A. To prove this by indirect proof, use the following steps:

1. Assume Sally made a B in at least one course.
2. Since one requirement for making the all A honor roll is to have no grade lower than an A, this contradicts the fact that Sally did make the honor roll.
3. Therefore, the original supposition (1) is false and Sally made only As.

AREA: INTRODUCTION TO CONCEPTS OF GEOMETRY

OUTCOME: 8. Demonstrate the ability to solve algebraic equations.

NOTES FOR THE TEACHER

This outcome is covered in the Algebra I Curriculum Guide, Area: Simple Equations, Inequalities, and Problems, pages 61-79.

AREA: INTRODUCTION TO CONCEPTS OF GEOMETRY

OUTCOME: 9. Use a compass and straightedge to make geometric constructions including parallel lines, perpendicular lines, and equal line segments.

NOTES FOR THE TEACHER

Prerequisites for Students

1. How to use a compass.
2. How to copy an angle.
3. How to bisect a line segment.
4. How to bisect an angle.

Steps to Success

Give the students an opportunity to practice with the tools. Have them use the compass to draw a circle; then have them draw a smaller or larger circle using the same center. Ask them to give a name to this set of circles. Have the students use their straightedge to draw an angle of any measure on their paper; then have them copy this angle. Have the students use their straightedge to draw a line segment; then use their compass to make an equilateral triangle of sides equal to the original segment. Have them fold the paper and compare the size of the angles of the equilateral triangle.

ACTIVITY 1 - OUTCOME 9

Through a point "p" not on a given line "ℓ", construct a line parallel to the given line.

Given: Line "ℓ" and point "p" not on "ℓ".

To construct: Using a compass and a straightedge, construct a line through "p" that is parallel to "ℓ".

ACTIVITY 1 - OUTCOME 9 (Cont'd)

Hint: Begin by drawing a line through "p" that intersects "ℓ".

Figure 1

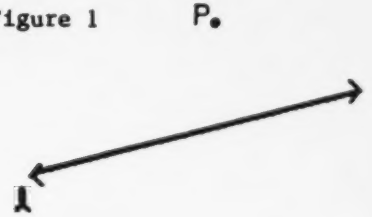
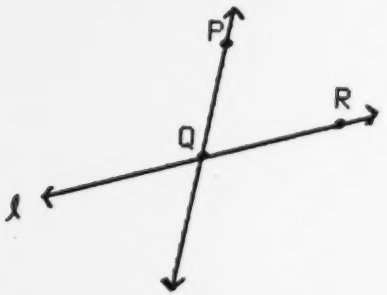


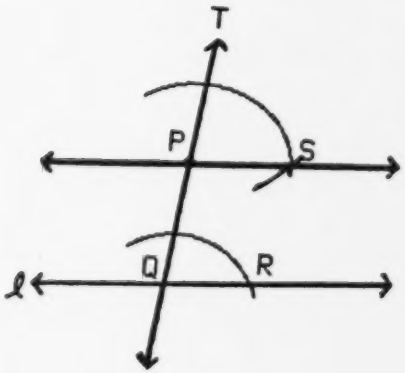
Figure 2



Process: In figure 2, copy $\angle PQR$ at P with $\overrightarrow{QP}$ being one side of the angle and P being the vertex of the angle. The other side of the angle $\overrightarrow{PS}$ will be parallel to "ℓ".

Proof: $\angle PQR \cong \angle TPS$ by construction

$\overrightarrow{PS} \parallel \ell$ because, if two lines in the same plane are cut by a transversal so that their corresponding angles are equal, the lines are parallel.



AREA: INTRODUCTION TO POINTS, LINES, AND PLANES

NOTES FOR THE TEACHER

This area is covered in the Area: Introduction to Concepts of Geometry.

AREA: INTRODUCTION TO ANGLES

OUTCOMES:
1. Apply the protractor theorem to the measurement of angles and use measurement to determine congruence.
2. Classify angles by types, according to measurement.
3. Describe special relationships between pairs of angles: i.e., (1) adjacent angles, (2) vertical angles, (3) complementary angles, (4) supplementary angles, (5) corresponding angles (reference parallel lines), (6) alternate interior angles of parallel lines.

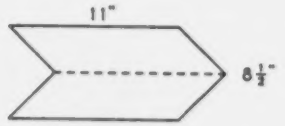
NOTES FOR THE TEACHER

Prerequisites for Students

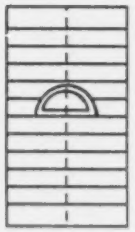
1. Know the protractor theorem, and have a protractor, paper and pencil.
2. Understand the Euclidean concept of parallel lines and the definition of transversal.

ACTIVITY 1 - OUTCOME 1

1. Have the students fold a sheet of lined paper in half, lengthwise.



Open the paper and lay it flat. Place the base of the protractor on any line, with the focal point of zero on the fold. Point out that the crease and any of the horizontal lines on the paper form 90° angles.



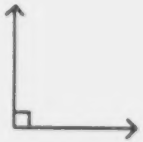
ACTIVITY 1 - OUTCOME 1 (Cont'd)

2. Emphasize that all angles of exactly 90° are congruent to each other. Point out some 90° angles in the room (floor tiles, wall blocks, windows, doors, corners of paper, books). Have students measure various items to determine whether they actually measure 90°.

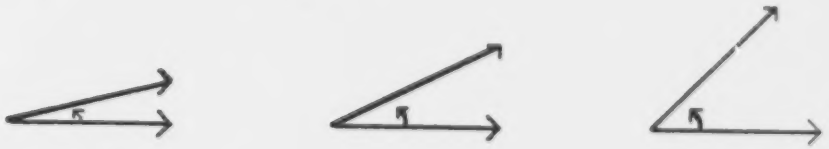
ACTIVITY 2 - OUTCOME 2

Discuss the following definitions.

1. Angles with measures of 90° are called right angles. When right angles appear in the book, they will be indicated by a small box-shaped figure.



2. Acute angles have measures less than 90°. Another definiton of acute is having a sharp or keen point. This definition may help students remember that acute refers to angles less than 90°.



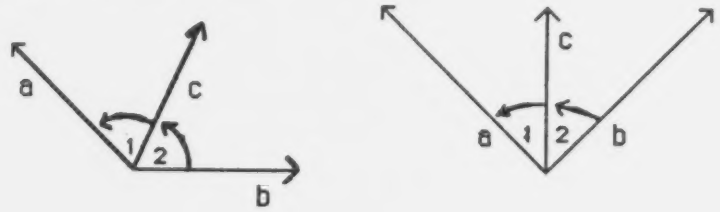
ACTIVITY 2 - OUTCOME 2 (Cont'd)

3. Obtuse angles have measures greater than 90° but less than 180°. Another definition of obtuse is blunt, dull, or thick. This definition may help students remember that obtuse angles are larger or more spread out than acute angles.

ACTIVITY 3 - OUTCOME 3

Relate the special relationships between pairs of angles that will be used in geometry to definitions that the students already know from everyday life.

a. Adjacent means "next to" each other. Adjacent classrooms share a wall. Adjacent neighbors in houses share a fence or property line. In condos or apartments, walls are shared. Adjacent angles share a common side.

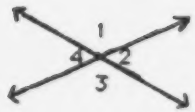


In both diagrams, ∠1 is adjacent to ∠2, and ∠1 and ∠2 share side c. Adjacent angles are two angles in the same plane that have a common vertex and a common side but have no interior points in common.

ACTIVITY 3 - OUTCOME 3 (Cont'd)

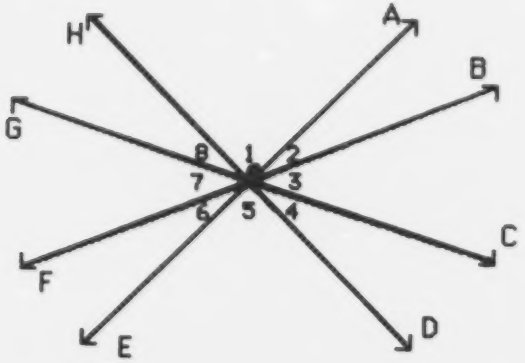
b. Vertical means straight up and down.

In this picture, ∠3 is "straight down" from ∠1. ∠s 1 and 3 are said to be vertical angles.



If the picture is turned sideways, ∠4 is "straight up" from ∠2.

These are also vertical angles. Vertical angles are "OPPOSITE" one another, and are formed by <u>pairs of opposite rays</u>.



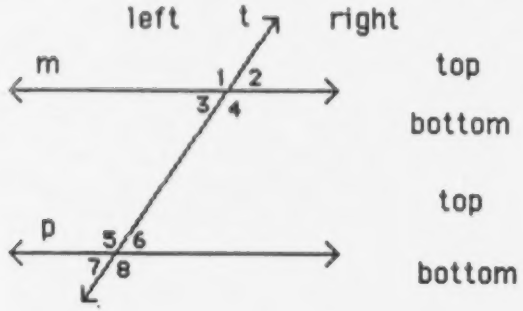
Use this diagram.

Rays OA and OE are opposite rays, OB and OF are also opposite rays; therefore, ∠2 and ∠6 are vertical angles. Find the other pairs of vertical angles using opposite rays.

c. Complementary angles are <u>two</u> angles whose sum equals 90°. Stress that <u>only</u> <u>two</u> angles are considered to be complements of one another. To find a complement, subtract the measure of the given angle from 90°. The complement of a 35° angle is (90 - 35) = 55°.

d. Supplementary angles are <u>two</u> angles whose sum equals 180°. Again, <u>only</u> <u>two</u> angles are supplementary. Also point out that supplementary and complementary angles do not have to be adjacent, in the same picture or in the same plane. To find the supplement of an angle, subtract the measure of the given angle from 180°. The supplement of a 65° angle is (180 - 65) = 115°.

e.



Use this diagram for e and f.

In the diagram above - lines m and p are parallel. Line t is a transversal.
NOTE: Lines m and p have a top and bottom. Line t has a left and right.

Corresponding angles are formed by parallel lines and a transversal. They have the same relative position on both parallel lines.

∠2 is on the top of line m and to the right of t.

∠6 is on the top of p and to the right of t.

ACTIVITY 3 - OUTCOME 3 (Cont'd)

Therefore, ∠s 2 and 6 are corresponding angles.

Find 3 different pairs of corresponding angles.

f. Point out that the space between line m and p could be considered the "interior" of those two lines.

"Alternate" means on opposite side of the transversal (t); therefore, ∠3 and ∠6 are alternate interior angles. Find another pair of alternate interior angles.

NOTE: Alternate exterior angles are not defined but can be related through the use of the definitions of vertical and corresponding angles.

AREA: SPECIAL RELATIONS OF LINES AND PLANES

OUTCOMES: 1. Develop deductive proofs concerning parallel lines.
2. Develop deductive proofs concerning perpendicularity of lines and planes.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know theorems and corollaries pertaining to parallel lines.
2. Know how to recognize corresponding angles, alternate interior angles, etc.

Steps to Success

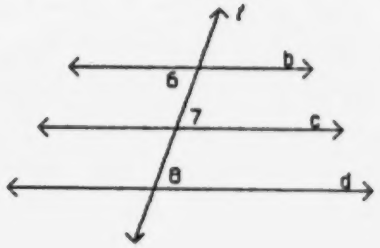
It is imperative that students understand and be able to communicate ideas relative to parallel lines because these concepts form the basis of future work.

ACTIVITY 1 - OUTCOME 1

Given: The plane figure;

$\angle 6 \cong \angle 8$; $c \parallel d$.

Prove: $b \parallel c$.



ACTIVITY 1 - OUTCOME 1 (Cont'd)

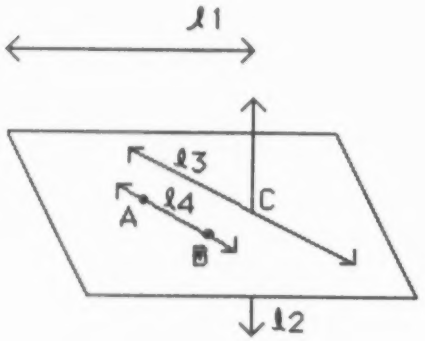
Make an audio cassette tape with a step by step explanation of the figure, the problem, and the proof. The tape should include detailed instructions for the students to know when to turn the tape off or on to allow them to discuss their method of proof, to write their proof, and to check their proof with the tape recording.

Send the students in small groups to the activity center where the tape player, tape, and activity sheet are located. Have printed instructions at the table. Designate a group leader to compile suggestions and to record the group's decisions about the proof.

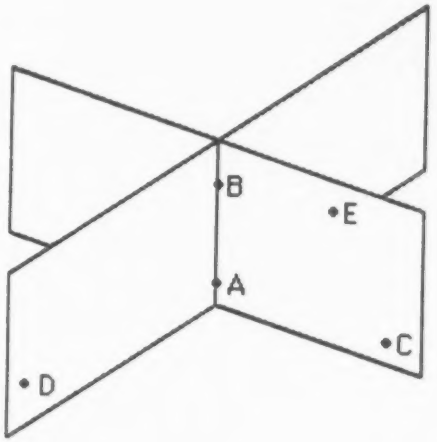
ACTIVITY 2 - OUTCOMES 1 and 2

Make transparencies or activity sheets similar to the ones below.

RELATIONSHIPS OF LINES AND A PLANE



TWO INTERSECTING PLANES



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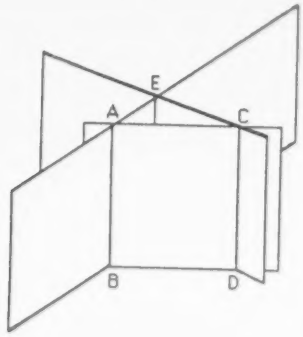
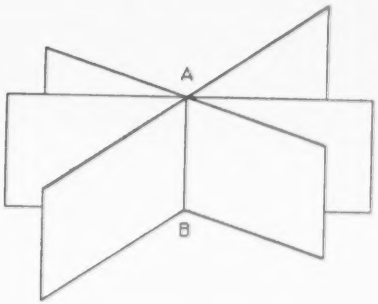
67

INTERSECTION OF THREE PLANES



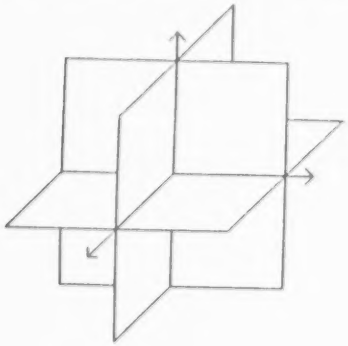
NO COMMON INTERSECTION

INTERSECTION OF THREE PLANES



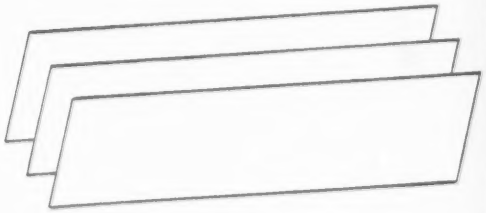
COMMON LINE

INTERSECTION OF THREE PLANES



ONE POINT

INTERSECTION OF THREE PLANES = $\varnothing$



PLANES COINCIDE

ACTIVITY 2 - OUTCOMES 1 and 2 (Cont'd)

Use the previous figures and have students discuss and answer questions similar to the ones below.

1. What is the relationship between ℓ_1 and the plane?
2. What is the relationship between ℓ_3 and the plane?
3. What is the relationship between ℓ_2 and the plane?
4. What is the relationship between ℓ_1 and ℓ_4 ?
5. What is the relationship between ℓ_3 and ℓ_4 ?
6. What is the relationship between ℓ_2 and ℓ_3 ?
7. If ℓ_2 is perpendicular to the plane, what is the relationship between ℓ_2 and ℓ_3 ? What is point C called?
8. What is the result when two planes intersect?
9. What are the possible outcomes if three planes intersect?

AREA: TRIANGLES

OUTCOMES:
1. Classify triangles according to sides.
2. Classify triangles according to angles.
3. Use the postulates of congruence to prove triangles congruent.
4. Prove segments and angles congruent by proving that they are corresponding parts of congruent triangles.
5. Prove triangles similar.
6. Use the corresponding sides of similar triangles to set up proportions.
7. Solve proportions.
8. Prove that the sum of the angles of a triangle equals 180 degrees.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know various types of triangles.
2. Know postulates and theorems of congruence.
3. Know postulates and theorems of similarity.
4. Know that the sum of the angles of a triangle is 180°.

Steps to Success

Present the following definitions:

equilateral triangle - has three equal sides
isosceles triangle - has a pair of equal sides
scalene triangle - has no equal sides (or angles)
right triangle - has one right angle
obtuse triangle - has one obtuse angle
acute triangle - has three acute angles
equiangular triangle - has three equal angles

ACTIVITY 1 - OUTCOMES 1 and 2

Identify each triangle as scalene, isosceles, or equilateral.

1.

2.

3.

4.

Identify each triangle as acute, right, or obtuse.

5.

6.

7.

8.

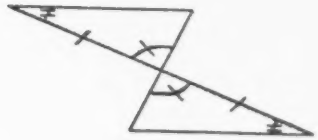
Answers

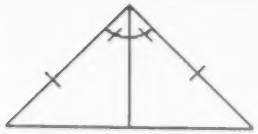
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____

ACTIVITY 2 - OUTCOMES 3 and 4

State which congruence postulate could be used to prove the triangles congruent.

1. 1._____

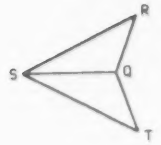
2.  2._____

3.  3._____

4. 4._____

Write a conclusion based on the given information.

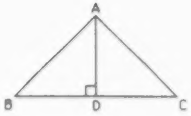
5. Given: $\overline{SQ}$ bisects $\angle RST$. 5._____



Therefore, _____.

6. Given: $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$. 6._____

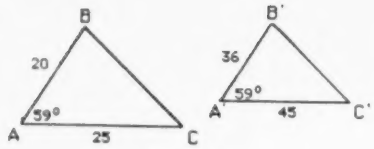
Therefore, $\overline{DC} \cong$ _____ and $\angle ADB \cong$ _____.



ACTIVITY 3 - OUTCOMES 5, 6, and 7

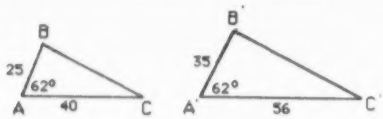
Example 1: Use the SAS Similarity Theorem to decide if

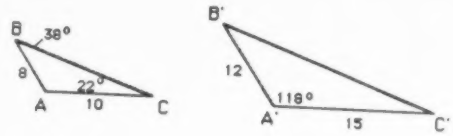
$\triangle ABC \sim \triangle A'B'C'$. $\angle A \cong \angle A'$ and $\frac{20}{36} = \frac{25}{45}$; yes.

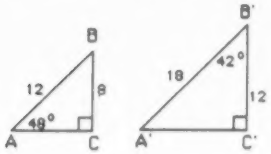


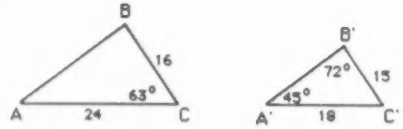
Solution: Since $\angle A \cong \angle A'$; side AB and A'B' and side AC and A'C' are in proportion, $\triangle ABC \sim \triangle A'B'C'$ by SAS.

Use the SAS Similarity Theorem to decide if $\triangle ABC \sim \triangle A'B'C'$.

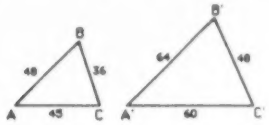
1. 

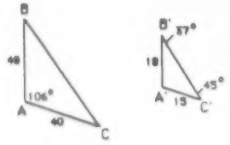
2. 

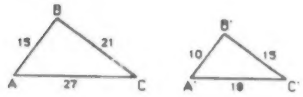
3. 

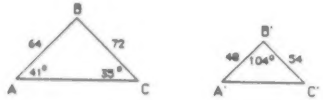
4. 

Decide if $\triangle ABC \sim \triangle A'B'C'$. State the theorem used.

5. 

6. 

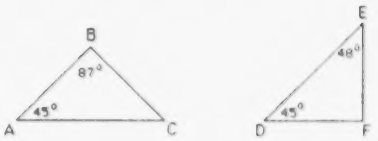
7. 

8. 

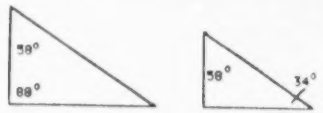
ACTIVITY 3 - OUTCOMES 5, 6, and 7 (Cont'd)

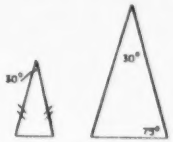
Example 2: Find the missing angle measures. Is $\triangle ABC \sim \triangle DEF$?

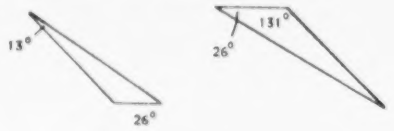
Solution: $m\angle C = 180 - (87 + 45) = 48$
$m\angle F = 180 - (45 + 48) = 87$
$\triangle ABC \sim \triangle DFE$



Find the missing angle measures. Tell if each pair of triangles is similar.

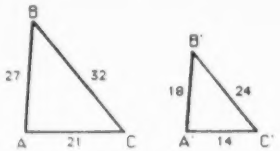
1. 

2. 

3. 

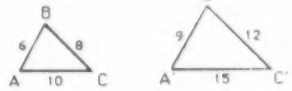
Example 3: Use the SSS Similarity Theorem to decide if $\triangle ABC \sim \triangle A'B'C'$.

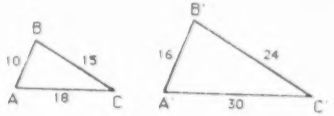
$$\frac{27}{18} = \frac{32}{24} = \frac{21}{14}$$

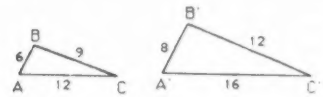


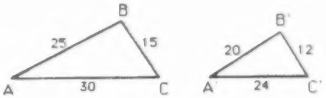
Solution: The three sides are not in proportion; therefore, $\triangle ABC$ is not similar to $\triangle A'B'C'$.

In the following examples, use the SSS Similarity Theorem to decide if $\triangle ABC \sim \triangle A'B'C'$.

1. 

2. 

3. 

4. 

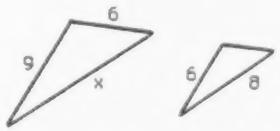
ACTIVITY 3 - OUTCOMES 5, 6, and 7

Example 3: The triangles are similar. Write a proportion and solve for x.

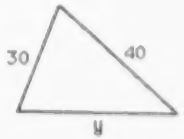
$\frac{9}{6} = \frac{x}{8}$

$6x = 72$

$x = 12$



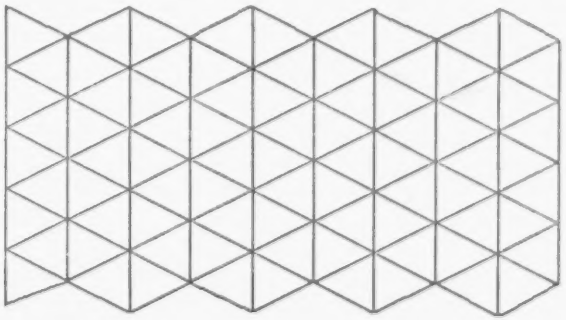
Write a proportion and solve for each length if the triangles are similar.



x = _____
y = _____

ACTIVITY 4 - OUTCOME 8

Using a grid of equilateral triangles, like the one below, locate any point. The sum of the angles about a point is 360°. There are six equal angles; therefore, each angle is 60°. Since the triangles are equilateral, they are also equiangular and all angles would measure 60°. The sum of the angles of any of these triangles is, therefore, 180°.



81

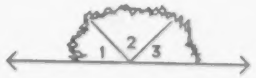
82

ACTIVITY 5 - OUTCOME 8

Students should be reminded that a straight line contains 180°.

Students need pencil, paper, straightedge, and protractor.

Have the students use their straightedge to construct a triangle (any triangle). Have them shade in the angles at the 3 vertices and tear off each corner and put the vertices together along the straightedge to form a straight line.



Therefore, m 1 + m 2 + m 3 = 180°.

83

84

AREA: TRIANGLES

OUTCOMES: 3. Use the postulates of congruence to prove triangles congruent.
4. Prove segments and angles congruent by proving that they are corresponding parts of congruent triangles.

NOTES FOR THE TEACHER

The understanding of the word "congruence" is essential. A simple definition that often works is "exactly equal in measure."

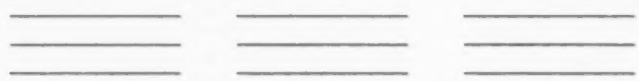
ACTIVITY 1 - OUTCOMES 3 and 4

Cut out two congruent triangles and show them on the overhead, a bulletin board, or a felt board. Display them so they do not look congruent.

Lift the triangles and place them so they are exactly on top of each other. Line up each angle and side to show they are "exactly the same size."

ACTIVITY 2: OUTCOMES 3 and 4

Take some straws, toothpicks, pick-up sticks, or dry pine needles. Cut them into three segments. The segments do not have to be equal, but each student's set should be from the same substance.



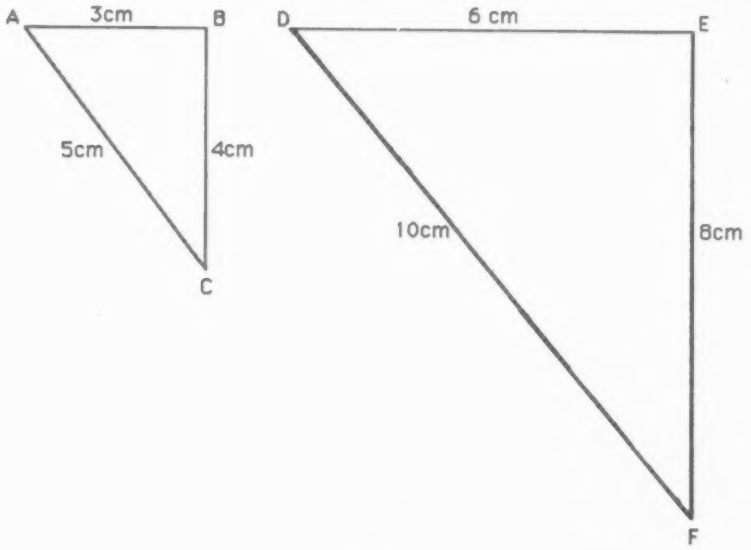
Have each student form a triangle with the pieces. Compare triangles with other students'. When different triangles match angle for angle and side for side, they are congruent.

AREA: TRIANGLES

OUTCOMES:
5. Prove triangles similar.
6. Use the corresponding sides of similar triangles to set up proportions.
7. Solve proportions.

NOTES FOR THE TEACHER

Define similarity as having congruent angles and proportional sides. Identify corresponding parts by demonstrating that the congruent angles must coincide, and that the corresponding sides are those opposite equal angles.



ACTIVITY 1 - OUTCOMES 5, 6, and 7

Give each student a copy of the triangles on the previous page. Have the students cut them out and place $\angle B$ on top of $\angle E$, $\angle A$ on top of $\angle D$, $\angle C$ on top of $\angle F$, until the students are satisfied that the angles match exactly. The students can easily see that these two triangles have three congruent angles but that the sides are obviously not congruent.

Check for proportionality by setting up ratios of the corresponding sides. This can be done in a number of ways, but an easy way is to put the sides of the smaller triangle over the sides of the larger triangle.

(1) $\frac{\text{Smaller Triangle}}{\text{Larger Triangle}}$ $\quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

(2) Substitute the value for each side $\quad \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$

(3) Check for proportionality by reducing to equal ratios $\quad \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

or

by finding cross products

$\frac{3}{6} = \frac{4}{8}$ $\quad 3 \cdot 8 = 4 \cdot 6$ $\quad 24 = 24$

$\frac{4}{8} = \frac{5}{10}$ $\quad 4 \cdot 10 = 8 \cdot 5$ $\quad 40 = 40$

$\frac{3}{6} = \frac{5}{10}$ $\quad 3 \cdot 10 = 6 \cdot 5$ $\quad 30 = 30$

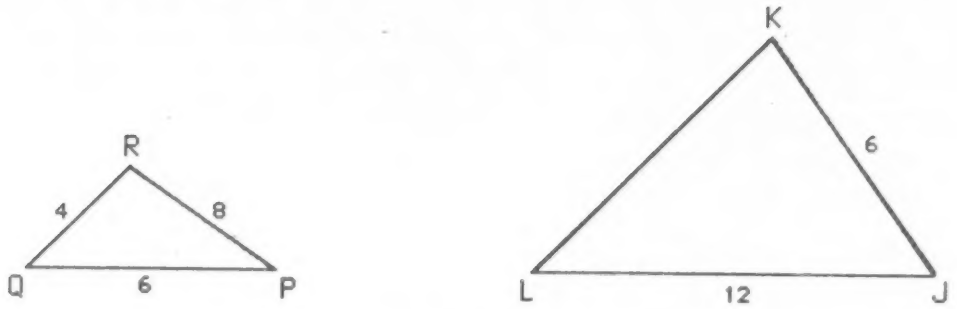
NOTE: A corresponding ratio reduced to lowest terms is called the "scale factor."

(4) Add the sides of the smaller triangle, 3 + 4 + 5 = 12. This is the perimeter. Now use proportions to find the perimeter of the larger triangle. Start with the "scale factor" of the two traingles. Write the perimeter of the smaller triangle over the perimeter of the larger triangle in both ratios, the order is important.

$$\frac{1}{2} = \frac{12}{P}$$
$$1p = 24$$
$$p = 24$$

Therefore, the perimeter of the larger triangle is 24.

ACTIVITY 2 - OUTCOMES 6 and 7



The two triangles are similar. Identify the corresponding congruent angles and the corresponding proportional sides. Find the scale factor, and determine the length of the missing side KL.

AREA: TRIANGLES

OUTCOMES: 9. Solve problems using the sum of angles theorem and its corollaries.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Know how to solve a linear equation with one variable.

Step to Success

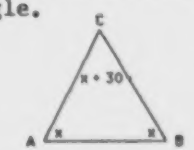
The use of manipulatives will enhance the study and understanding of this area.

ACTIVITY 1 - OUTCOME 9

Use the sum of angles theorem (the sum of the angles of a triangle is 180°) to solve problems involving angle measures of triangles.

Example 1: The vertex angle of an isosceles triangle exceeds each base angle by 30°. Find the number of degrees in each angle of the triangle.

Solution: Let x = the number of degrees in each base angle.



Then x + 30 = the number of degrees in the vertex angle.

$$\begin{aligned} x + x + x + 30 &= 180 \\ 3x + 30 &= 180 \\ 3x &= 150 \\ x &= 50 \\ x + 30 &= 80 \end{aligned}$$

The three angles are 50°, 50°, and 80°.

AREA: TRIANGLES

OUTCOMES: 10. Know the Pythagorean Theorem.
11. Apply the Pythagorean Theorem to the solution of problems.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know how to square numbers and simplify radical numbers.
2. Know the definition of a right triangle and be able to identify correctly the legs and the hypotenuse.

A ball of cord or knitting yarn at least 100 feet long is necessary for these activities. Have a student tie a knot in the cord every 18 inches.

ACTIVITY 1 - OUTCOMES 10 and 11

Select three students to stand and represent the vertices of a right triangle. Give one student the yarn and ask him/her to hold the cord at one of the knots near the end of the cord. The second student is to hold the cord at the knot which is the third knot from the first student's. The third student is to hold the cord at the knot which is the fourth knot from the second student's. Get a fourth student to hand the cord back to the first student and remind the students that they represent the vertices of a right triangle. How many knots does it take to go from the first student to the second? (3 knots) How many knots does it take to go from the second student to the third? (4 knots) How many knots does it take to go from the third student back to the first? (5 knots) The teacher should write 3, 4, 5 on the board.

Follow the same procedure with students holding knots 6 and 14. How many knots does it take to get back to the first student? What are the lengths of the sides of the triangle? 6, 8, 10. Write 6, 8, and 10 on the board.

Continue this process using triangles of the following sides: 5, 12, 13; 7, 24, 25; 8, 15, 17.

NOTE: Each time the students should determine the length of the hypotenuse, i.e. the number of knots required to go from the third student back to the first student.

Ask the students if they can see any relationships between the sides of the triangles. This should lead students to discover that in a right triangle the square of the hypotenuse is equal to the sum of the squares of the two legs. This is the Pythagorean Theorem stated algebraically $a^2 + b^2 = c^2$.

Use the Pythagorean Theorem to verify that the triangle with sides of 3, 4, and 5 is a right triangle.

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

Show that a triangle with sides of 6, 8, and 11 cannot be a right triangle.

Use the Pythagorean Theorem to verify that 6, 8, 11 is not a right triangle.

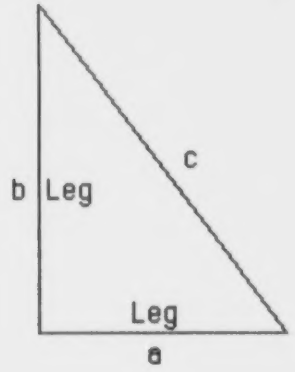
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = 11^2$$
$$36 + 64 = 121$$
$$100 \neq 121$$

Therefore the triangle is not a right triangle.

ACTIVITY 2 - OUTCOMES 10 and 11

Present the Pythagorean Theorem, demonstrate the theorem, and use the theorem to find unknown sides of the right triangle.

THE PYTHAGOREAN THEOREM

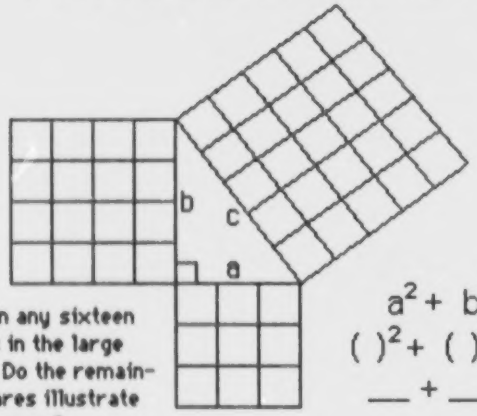


$$c^2 = a^2 + b^2$$

"The square of the hypotenuse of a right triangle equals the sum of the squares of the legs."

Demonstration of

PYTHAGOREAN THEOREM



Shade-in any sixteen squares in the large square. Do the remaining squares illustrate this theorem?

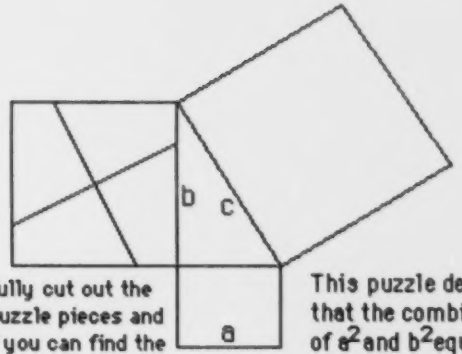
$a^2 + b^2 = c^2$

$(\ )^2 + (\ )^2 = (\ )^2$

$__ + __ = __$

$$a^2 + b^2 = c^2$$

The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.



Carefully cut out the five puzzle pieces and see if you can find the solution.

This puzzle demonstrates that the combined areas of a^2 and b^2 equal the area of c^2.

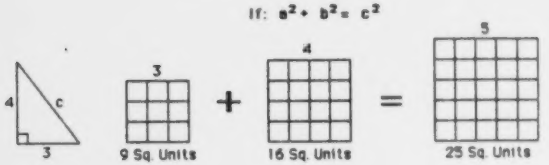
In the year 1830, Henry Perigal, a London stockbroker, discovered this unique dissection. The four shapes together with the small square can be filled into the large square.

Verification

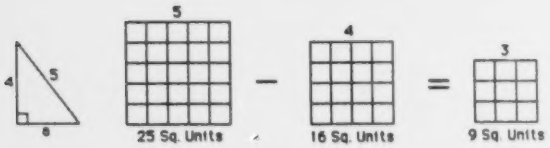
THE PYTHAGOREAN THEOREM

To find the length of an unknown leg given one leg and the hypotenuse.

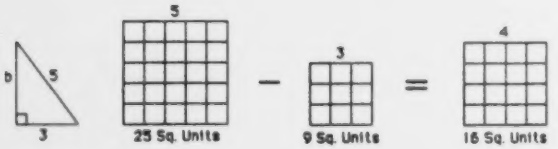
If: $a^2 + b^2 = c^2$



Then: $c^2 - b^2 = a^2$



And: $c^2 - a^2 = b^2$



Application

AREA: APPLICATION OF CONGRUENT TRIANGLES

OUTCOME: 1. Apply the theorems of congruent triangles to the proof of theorems concerning parallelograms, rectangles, rhombuses, trapezoids, and other polygons.

NOTES FOR THE TEACHER

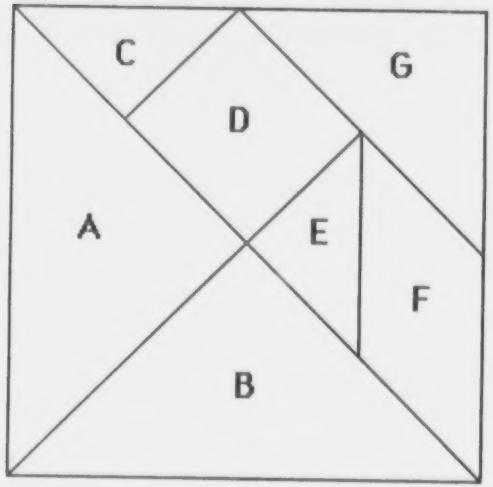
Use inductive reasoning to determine the properties of convex quadrilaterals. Use tangram pieces to create the quadrilaterals.

Materials needed: Copies of the tangram square, scissors, ruler, protractor, and compass.

ACTIVITY 1 - OUTCOME 1

Give each student a copy of the tangram and have him/her cut out the pieces.

(1) Fit the pieces together, one at a time, two at a time, three at a time, etc., to form convex polygons. (Students may work individually or in small groups.)
(2) Name the shape formed by the pieces.
(3) Use a compass, ruler, and/or protractor to give the properties of special polygons-- equilaterial triangle, isosceles triangle, parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid. Students may complete a chart similar to the one that follows.



PROPERTIES OF QUADRILATERALS

Use tangram pieces, a ruler, and a protractor to complete this table.

Property	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles trapezoid
Both pairs of opposite sides are parallel.						
Exactly one pair of opposite sides is parallel.						
Both pairs of opposite sides are congruent.						
Exactly one pair of opposite sides is congruent.						
All sides are congruent.						
Opposite angles are congruent.						
All angles are right angles.						
Diagonal forms two congruent triangles.						
Diagonal forms two congruent isosceles triangles.						
Diagonals are congruent.						
Diagonals bisect the opposite angles.						
Diagonals are perpendicular.						
Diagonals bisect each other.						

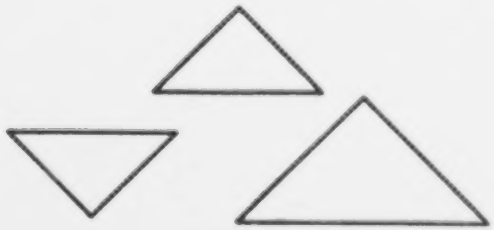
ACTIVITY 2 - OUTCOME 1

Fit the pieces of the tangram together to form concave polygons.

ACTIVITY 3 - OUTCOME 1

Place the three triangles inside each of the quadrilaterals. The triangles can be so arranged as to make a figure congruent with each of the quadrilaterials.

With these three pieces,



make these two shapes.

AREA: APPLICATION OF CONGRUENT TRIANGLES

OUTCOMES 2. Prove triangle inequality theorems.
3. Apply triangle inequality theorems to the solution of problems.

NOTES FOR THE TEACHER

Steps to Success

Some advance preparation is necessary. One way to demonstrate the triangle inequality theorems is with a felt board and some strips of felt cut into various marked lengths. Another way is to have each student see the relationships for himself by using strips of paper, cardboard or wood cut in the following lengths.

two - 3-unit strips
two - 4-unit strips
one - 5-unit strip
one - 6-unit strip
one - 8-unit strip

The units can be any consistent unit of measure: inches, centimeters, or 2-inch units.

ACTIVITY 1 - OUTCOMES 2 and 3

1. Try to make a triangle by using two 3-unit strips and an 8-unit strip. What happens?

2. Substitute a 4-unit strip for one of the 3-unit strips. What occurs?

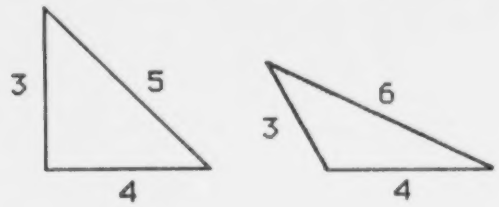
3. Try the two 4-unit strips with the 8-unit strips. Can you make a triangle yet?

This illustrates that the sum of the lengths of two sides of a triangle must always be greater than the third side.

ACTIVITY 1 - OUTCOMES 2 and 3 (Cont'd)

4. Use the strips to form the following triangles simultaneously:

   3, 4, 5 units and 3, 4, 6 units



   Use the protractor to measure the angles where the 3 and 4 unit strips are joined.

   This leads directly to the theorems: If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second. If two sides of one triangle are congruent to two sides of a second triangle, but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is larger than the third side of the second.

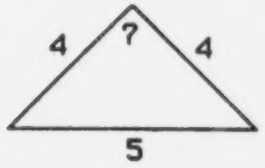
5. Make a triangle using the 4, 5, and 8 unit strips. What type angle is formed at the point where the 4 and 5 unit strips meet?

It is an obtuse angle. Notice that the obtuse angle is opposite the side that is 8 units long. How does this triangle look if we apply the Pythagorean rule and let the longest side be "c"? (Remind the students that the hypotenuse is always the longest side of a right triangle.)

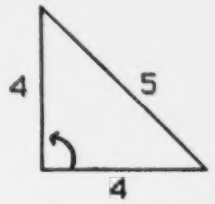
$$\begin{array}{rcl} 4^2 + 5^2 & \square & 8^2 \\ 16 + 25 & \square & 64 \\ 41 & < & 64 \end{array}$$

NOTE: The square of the longest side is much larger than the sum of the squares of the shorter sides, and the longest side is opposite the largest angle.

6. Create a triangle by using the two 4-unit strips and the 5-unit strip.



What type angle occurs where the two 4-unit strips join? (It should be obvious that the angle is acute. If it is not obvious, have the students arrange their triangle like the one below so they will be able to tell that the angle created where the two 4-unit strips join is less than 90° or acute.



Look at this with the Pythagorean Rule.

$$4^2 + 4^2 \square 5^2$$
$$16 + 16 \square 25$$
$$32 > 25$$

NOTE: The square of the longest side is smaller than the sum of the squares of the two shorter sides and the angle opposite the longest side is smaller than 90°, or acute.

AREA: SIMILAR POLYGONS

OUTCOME: 1. Know the definitions and properties of similar polygons.

NOTES FOR THE TEACHER

The students will use tangram pieces to form geometric shapes and then determine the area of each shape. (Tangram templates can be purchased at most school supply stores or can be made using the pattern on page 51 or 58 of this book.)

ACTIVITY 1 - OUTCOME 1

Give each student a set of tangram pieces to be used to answer the following questions.

1. Which of the pieces is congruent to A?
2. Which pieces are similar?
3. Which pieces contain at least one right angle?

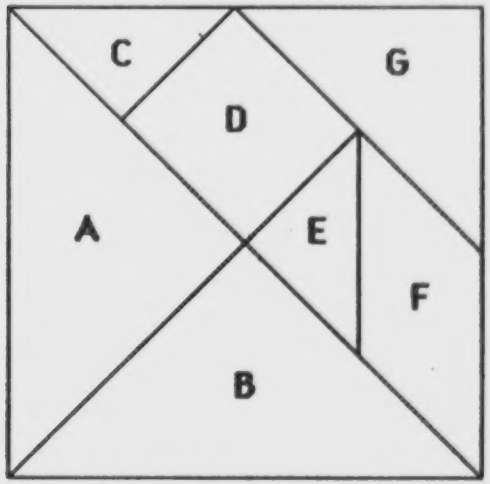
For 4, 5, 6, & 7, trace the smaller figure in the larger figure.

4. How many Es are in D? (Write a formula for the area of D in terms of E.)
5. How many Cs are in G? (Write a formula for the area of G in terms of C.)
6. How many Ds are in A? (Write a formula for the area of A in terms of D.)
7. How many Es are in B? (Write a formula for the area of B in terms of E.)
8. Make an isosceles trapezoid using C, D, and E. How many Cs cover the trapezoid? You may say, "The area of the trapezoid is_____ C units where C is the unit of area."
9. Make a trapezoid using D, E, and F. How many Cs cover the trapezoid; i.e., what is the area?
10. How does the area of the trapezoid formed with C, D, and E compare with the trapezoid formed using D, E, and F?

ACTIVITY 1 - OUTCOME 1 (Cont'd)

11. Use E as the unit of area. What is the area of B?
12. Use E as the unit of area. What is the area of the triangle formed by A and B?
13. Form a trapezoid using E and F. How many Es cover the trapezoid?
14. Form a trapezoid using C and D. Compare the area of the trapezoid with the trapezoid in problem 13.
15. Have students write questions about the areas of figures formed by the tangram pieces.

Carefully cut out the tangram below.



AREA: SIMILAR POLYGONS

OUTCOME: 2. Apply the definitions and properties of similar polygons to the proof of other theorems and the solution of problems.

NOTES FOR THE TEACHER

It is often necessary to review ratio and proportion before beginning the study of similar polygons.

One way to review ratio is to use the ratios of parts of the body. The teacher should be very cognizant of "special" physical characteristics such as obesity or handicaps.

Materials needed: Ball of string.

ACTIVITY 1 - OUTCOME 2

Give each student a piece of string approximately one yard long. Each student should use the string to:

1. Find the circumference of the base of the thumb. Hold the end of the string at the base of the thumb with the index finger. Wrap the string around the thumb. Be sure to hold the point where the string comes back to the endpoint of the string.
2. Find twice the circumference of the thumb.
3. Use twice the circumference of the thumb to go around the wrist at the watch band area. What is the ratio of the circumference of the thumb to the circumference of the wrist? (2:1)
4. Find the circumference of the wrist.
5. Find twice the circumference of the wrist.
6. Use twice the circumference of the wrist to go around the neck at the collar area. What is the ratio of the circumference of the wrist to the circumference of the neck? (2:1)
7. Measure the circumference of the base of the knee using the circumference of the neck. What is the ratio? (1:1)
8. Find twice the circumference of the neck or knee.
9. Measure the circumference of the waist using the circumference of the neck. What is the ratio? (2:1)

ACTIVITY 1 - OUTCOME 2 (Cont'd)

After completing the measures, students should give the following ratios:

1. $\frac{\text{Circumference of the thumb}}{\text{Circumference of the neck}} \longrightarrow \frac{\text{C thumb}}{\text{C neck}}$

2. $\frac{\text{Circumference of the knee}}{\text{Circumference of the wrist}} \longrightarrow \frac{\text{C knee}}{\text{C wrist}}$

3. $\frac{\text{Circumference of the thumb}}{\text{Circumference of the waist}} \longrightarrow \frac{\text{C thumb}}{\text{C waist}}$

4. $\frac{\text{Circumference of the waist}}{\text{Circumference of the knee}} \longrightarrow \frac{\text{C waist}}{\text{C knee}}$

Which of the following are true?

5. $\frac{\text{C thumb}}{\text{C neck}} = \frac{\text{C wrist}}{\text{C neck}}$

6. $\frac{\text{C wrist}}{\text{C neck}} = \frac{\text{C neck}}{\text{C knee}}$

7. $\frac{\text{C wrist}}{\text{C neck}} = \frac{\text{C neck}}{\text{C waist}}$

8. $\frac{\text{C waist}}{\text{C knee}} = \frac{\text{C waist}}{\text{C neck}}$

AREA: SIMILAR POLYGONS

OUTCOMES: 1. Know the definitions and properties of similar polygons.
2. Apply the definitions and properties of similar polygons to the proof of other theorems and to the solution of problems.

NOTES FOR THE TEACHER

Prerequisite for Students

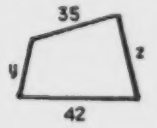
1. Know the properties of similar polygons.

Step to Success

Show the relationship between similar polygons and congruent polygons. Congruent polygons are similar polygons where the ratio of the corresponding sides is 1:1.

ACTIVITY 1 - OUTCOMES 1 and 2

The pairs of quadrilaterals below are similar. Find the measures of the indicated sides.



x = ____
y = ____
z = ____

a = ____
b = ____
c = ____

AREA: SIMILAR POLYGONS

OUTCOMES:
3. Know the right triangle definitions of the sine, cosine, and tangent functions.
4. Use the sine, cosine, and tangent functions to solve problems.
5. Apply the properties of the 30-60-90 degree triangle to determine the values of the sine, cosine, and tangent of 30-degree and 60-degree angles.
6. Apply the properties of the 45-45-90 degree triangle to determine the values of the sine, cosine, and tangent of 45-degree angles.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Must be able to set up and solve proportions.
2. Know the 45-45-90 and 30-60-90 right triangle relationships.
3. Know and be able to use the Pythagorean Theorem.

Steps to Success

Teach the trigonometric function relationships using the mnemonic device described below.

SOHCAHTOA (or Sock-ah-toe-ah for those not phonetically inclined).

Tell students that Sohcahtoa is the great Indian chief of trigonometry. The letters of his name give clues to the trigonometric functions and their definitions.

SOH - Sine is opposite over hypotenuse

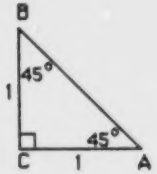
CAH - Cosine is adjacent over hypotenuse

TOA - Tangent is opposite over adjacent

Emphasize in the above definitions that the ratio being discussed is the length of the sides of the triangle. For example, sine is the ratio of the length of the opposite side over the length of the adjacent side.

ACTIVITY 1 - OUTCOMES 3, 4, and 6

Have the students develop the relationships of the 45-45-90 right triangle using the Pythagorean Theorem. Find the sine, cosine, and tangent of each angle using the SOHCAHTOA mnemonic device.



If a = 1 and b = 1, use the Pythagorean Theorem and find the length of c.

$$a^2 + b^2 = c^2$$
$$1^2 + 1^2 = c^2$$
$$2 = c^2$$
$$\sqrt{2} = c$$

Since $\angle A \cong \angle B$, $\triangle ABC$ is a 45-45-90 right triangle.

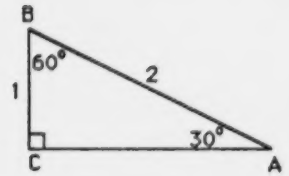
$$\sin A = \sin 45° \text{ (Remember SOH)} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos A = \cos 45° \text{ (Remember CAH)} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan A = \tan 45° \text{ (Remember TOA)} = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1$$

ACTIVITY 2 - OUTCOMES 3, 4, and 5

Have the students develop the relationships of the 30-60-90 right triangle using the Pythagorean Theorem. Then find the sine, cosine, and tangent of each angle using the SOHCAHTOA mnemonic device.



If a = 1 and c = 2, use the Pythagorean Theorem to find the length of b.

$a^2 + b^2 = c^2$

$1^2 + b^2 = 2^2$

$1 + b^2 = 4$

$b^2 = 3$

$b = \sqrt{3}$

sin A = sin 30° (Remember SOH) = $\frac{1}{2}$

cos A = cos 30° (Remember CAH) = $\frac{\sqrt{3}}{2}$

tan A = tan 30° (Remember TOA) = $\frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$

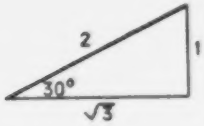
sin B = sin 60° (Remember SOH) = $\frac{\sqrt{3}}{2}$

cos B = cos 60° (Remember CAH) = $\frac{1}{2}$

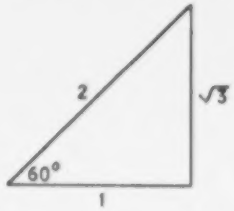
tan B = tan 60° (Remember TOA) = $\frac{\sqrt{3}}{1} = \sqrt{3}$

ACTIVITY 3 - OUTCOMES 3, 4, 5, and 6

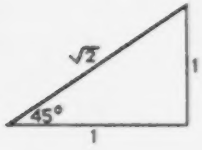
Show the students how to draw right triangles as described in Activities 1 and 2. Generate these values as needed rather than have the students memorize them. The values will be committed to memory with frequent use.



sin 30° =
cos 30° =
tan 30° =



sin 60° =
cos 60° =
tan 60° =



sin 45° =
cos 45° =
tan 45° =

AREA: SIMILAR POLYGONS

OUTCOMES: 7. Develop formulas for finding the area of polygonal regions.
8. Use formulas to find the area of polygons.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be familiar with finding the area of a rectangle.

   $A = bh$
   b = base
   h = height

   Using these terms will help in developing formulas for other polygonal regions.

2. Need pencil, paper, scissors, and a straight-edge.

ACTIVITY 1 - OUTCOMES 7 and 8

1. Create a parallelogram. Draw and cut a rectangle from a sheet of paper. Label the base and height.

Cut a triangular piece from one end of the rectangle. Place it on the other end of the rectangle.

The result is a parallelogram. How does the area of the parallelogram compare to the area of the original rectangle? What is the formula for the area of a parallelogram?

$A = bh$

2. Draw and cut a rectangle from a sheet of paper. Label the base and height. Cut the rectangle in half diagonally.

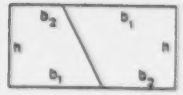
What is the result? The result is two congruent triangles. How does the area of each triangle compare to the area of the original rectangle? The area of one triangle is one half the area of the original rectangle.

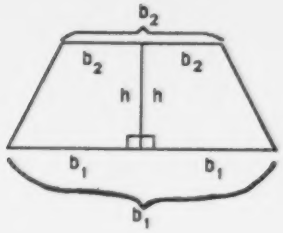
$A = \frac{1}{2}bh$

NOTE: At this point, it is good to stress the fact than <u>any</u> of the three sides of the triangle can be the base. The height or altitude is always the perpendicular distance drawn from the vertex opposite the base to the base. Have the students measure the other sides and the altitude. Compute the area. Point out that measuring inaccuracies may make small differences, but the new areas should approximate the original area.

3. Draw and cut a rectangle from a sheet of paper. Label the height and draw a slanted line between the two bases (which are equal).

ACTIVITY 1 - OUTCOMES 7 and 8 (Cont'd)

Label the two short pieces b_2, and the two longer pieces b_1.  Cut the rectangle into two pieces on the line. Turn one piece over and upside down to align the two b_1 pieces and the two b_2 pieces.



NOTE: This trapezoid should have the same surface area as the rectangle. The height has not changed, but the two parallel bases are different lengths. It is, therefore, necessary to find the average length of the bases. This can be done by adding the lengths of the two bases and dividing their sum by two.

$$A = \left(\frac{b_1 + b_2}{2}\right)h \qquad \text{or} \qquad A = \frac{1}{2}\left(b_1 + b_2\right)h$$

This average length of the bases is also the length of the median of the trapezoid.

Use these formulas in solving problems involving areas of polygons.

AREA: CIRCLES

OUTCOMES:
1. **Identify and define tangents, chords, secants, inscribed polygons, circumscribed polygons, central angles, inscribed angles, angles formed by a tangent and a chord, angles formed by two secants intersecting outside the circle.**
2. **Determine the measure of central angles, inscribed angles, angles formed by a tangent and a chord, angles formed by two secants intersecting inside the circle, and angles formed by two secants intersecting outside the circle.**
3. **Understand and work problems with concentric circles, intersecting circles, disjunct circles, internally tangent circles, and externally tangent circles.**

NOTES FOR THE TEACHER

Steps to Success

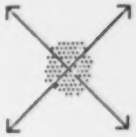
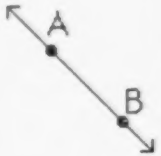
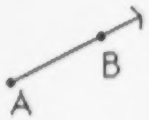
1. Present the definitions of tangent, chord, secant, inscribed polygons, circumscribed polygons, concentric circles, intersecting circles, internally tangent circles, externally tangent circles, and disjunct circles.
2. Several copies of the attached set of cards may need to be made. Remove any pairs that the students have not studied or that you do not wish reviewed. Add pictures and terms as needed.

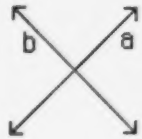
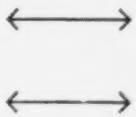
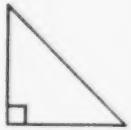
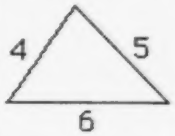
ACTIVITY 1 - OUTCOMES 1 and 3

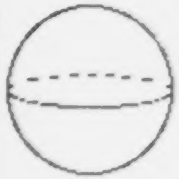
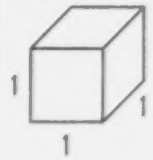
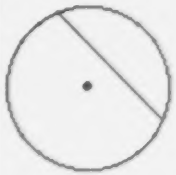
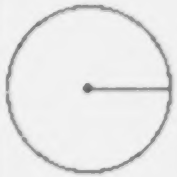
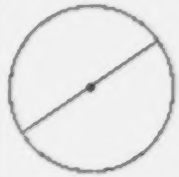
To review the definitions mentioned above and to promote student understanding, divide the students into small groups of 4 or 5 and let them play GEO-MAN.

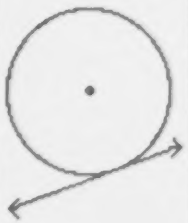
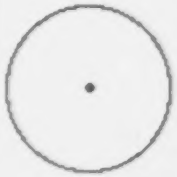
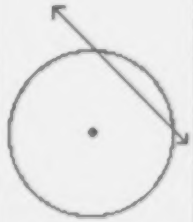
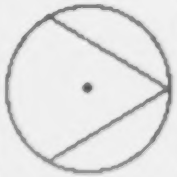
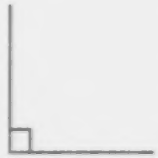
The attached game GEO-MAN, is played like OLD MAID. Deal all of the cards. Each player attempts to make "pairs" consisting of a picture and its identifying name. In turn, each player draws an unseen card from the hand of the person on his right.

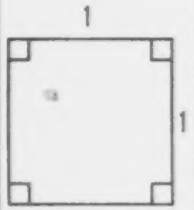
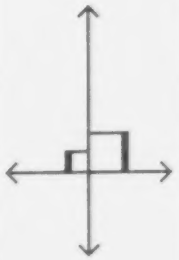
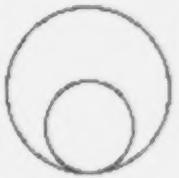
The person left with GEO-MAN is the loser. Sometimes, to make the game more intriguing, a picture or a name is intentionally omitted and the Wild Card is substituted. This matches anything but GEO-MAN. It must be used only as a last resort. If a player uses the Wild Card to match anything but the card with the missing pair, he is the loser instead of the GEO-MAN holder.

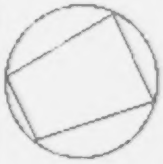
	ACUTE ANGLE		VERTICAL ANGLES		OBTUSE ANGLE		ADJACENT ANGLES
	LINE		POINT		SEGMENT		RAY

	Intersecting Lines		Parallel Lines		Equilateral Triangle		Isosceles Triangle
	Right Triangle		Scalene Triangle		CYLINDER		CONE

	SPHERE		PYRAMID		CUBE	WILD CARD	GEO MAN
	CHORD		CENTRAL ANGLE		RADIUS		DIAMETER

	TANGENT		CIRCLE		SECANT		INSCRIBED ANGLE
	HEXAGON		OCTAGON		PENTAGON		RIGHT ANGLE

	SQUARE		PARALLELOGRAM		Perpendicular Lines		RECTANGLE
	DISJUNCT CIRCLES		EXTERNALLY TANGENT CIRCLES		CONCENTRIC CIRCLES		INTERNALLY TANGENT CIRCLES

	Angle Formed By a Chord and a Tangent		Angle Formed By Two Secants
	INSCRIBED POLYGON		CIRCUMSCRIBED POLYGON

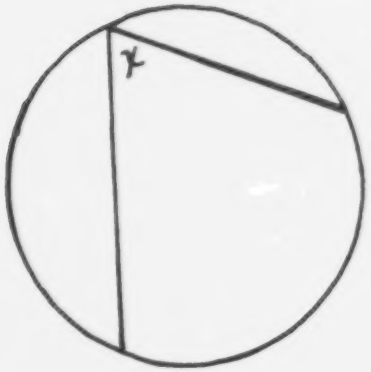
ACTIVITY 2 - OUTCOME 2

Students must understand definitions for arc, central angle, inscribed angles, tangent, chord, and secant. Also, the students must be able to correctly identify the intercepted arc(s).

Steps to Success:

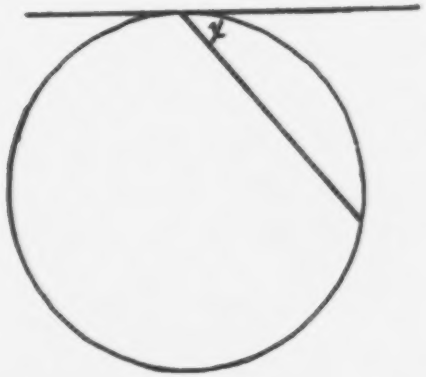
Draw the following pictures and make sure that students can correctly identify the chords, tangents, secants, and intercepted arcs.

Example 1:



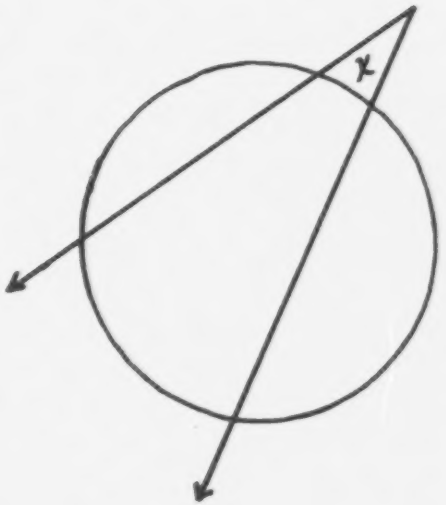
two chords

Example 2:



tangent and chord

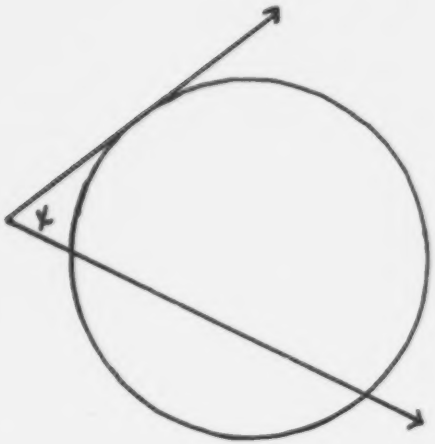
Example 3:



two secants

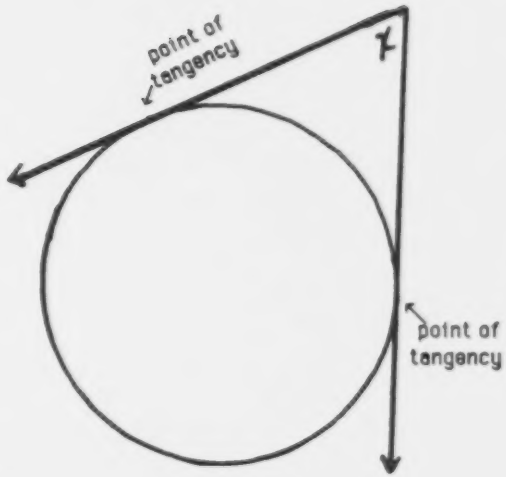
ACTIVITY 2 - OUTCOME 2 (Cont'd)

Example 4:



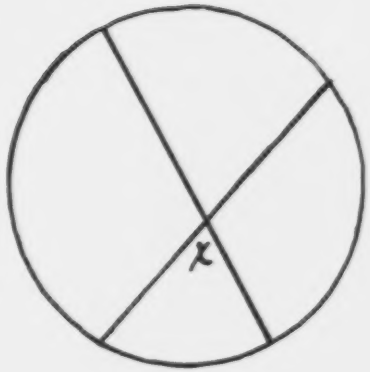
a secant and a tangent

Example 5:



two tangents

Example 6:



two chords

ACTIVITY 3 - OUTCOME 2

1. Have students identify the intercepted arcs for each example in Activity 2. Point out that for examples 1 and 2, there is only 1 intercepted arc, but for the rest, there are two.

2. Have the students describe the unknown angle (x) as <u>OUTSIDE</u> the circle or <u>INSIDE</u> the circle.

ACTIVITY 2 - OUTCOME 2 (Cont'd)

3.

If the unknown angle is:	its measure is:
OUTSIDE THE CIRCLE,	$\frac{1}{2}$ the difference of the intercepted arc.
INSIDE THE CIRCLE,	$\frac{1}{2}$ the sum of the intercepted arc.

4. Assign values to the arcs in each example and find the value of the angles.

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AREA: CIRCLES

OUTCOMES: 4. Find the circumference of a circle.
5. Find the area of a circle.

NOTES FOR THE TEACHER

These outcomes are covered in the General Mathematics Curriculum Guide. Outcome 4 is covered in Activity 2 on page 103 and Outcome 5 is covered in Activity 1 on page 125.

AREA: GEOMETRIC SOLIDS

OUTCOME: 1. Identify and use formulas to find the area and volume of cylinders, rectangular solids, pyramids, cones, and spheres.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Use area and perimeter formulas for polygons.
2. Use area and circumference formulas for circles.

Steps to Success

1. Spend time discussing the different types of geometric solids. Place emphasis on the various parts of the solids.
2. Area is "surface." Refer to area as "covering." Relate finding total or lateral surface area as wallpapering or covering with wrapping paper.
3. To wallpaper a room, the length and width of a wall are covered. This covers the surface area of the wall ($l \times w$) therefore, it involves two dimensions. All units of measure in the answers for area will be to the second power or squared and will be denoted by an exponent of 2. (Remember 2 dimensions, length and width.)
4. Refer to volume as "filling up." Filling a cooler with ice, a car with gas, a can with pop are ways volume is used daily. Some standard volume measures are quart, gallons, liters, ounces, and milliliters.
5. Volume uses 3 dimensions for measurement: length, width, and height (depth).

ACTIVITY 1 - OUTCOME 1

Have students construct a model of some of the regular polyhedrons: tetrahedrons, hexahedrons, dodecahedrons, icosahedrons, and octahedrons. Discuss and define edge, face, and vertex.

ACTIVITY 2 - OUTCOME 1

Have students prepare models which show the different parts of a cylinder and a prism. The following procedure can be used to prepare teaching models:

1. Cut two three-inch circles out of cardboard.
2. Use elastic dress maker thread to put the two circles together. The thread represents the elements of a cylinder and should be about four inches long. Be sure to put enough elements in to show a cylinder. (Space approximately 1/4 inch apart on each circle.)

3. Follow the same procedures in steps one and two to prepare models for a prism. Use cardboard triangles, squares, and octagons as bases.

To use the models, hold by the bases and stretch the elements until they are tight. Stretch to show right cylinders and prisms or oblique cylinders and prisms. Discuss the altitude of right and oblique cylinders and prisms.

Use the same models to show cones and pyramids. Stretch and twist, and use one portion to discuss the characteristics of the cone and the pyramid. Use both oblique and right solids.

ACTIVITY 3 - OUTCOME 1

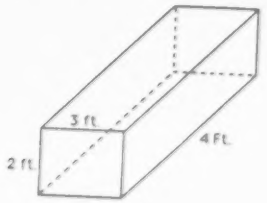
Use the textbook to assign problems involving the area and volume of the different solids. Be sure to include examples similar to the following:

ACTIVITY 3 - OUTCOME 1 (Cont'd)

Example 1: A barbeque grill was purchased as a gift and needs to be gift wrapped. Wrapping paper comes in 3 sizes:

25 square feet for $2.99
40 square feet for $4.99
60 square feet for $6.99

The box containing the grill measures 2 ft. x 3 ft. x 4 ft. Which amount of wrapping paper will cover the box?



Solution 1: In finding the area of the rectangular solid, it is important for students to realize that the area is made up of four lateral areas, two of which have measure 3 ft. x 4 ft., two of which have measure 2 ft. 4 ft., and two bases with measures 2 ft. x 3 ft. Find the total surface area by adding the areas of all sides.

(2 ft. x 4 ft.) x 2 = 16 sq. ft.
(3 ft. x 4 ft.) x 2 = 24 sq. ft.
(2 ft. x 3 ft.) x 2 = + 12 sq. ft.
52 sq. ft.

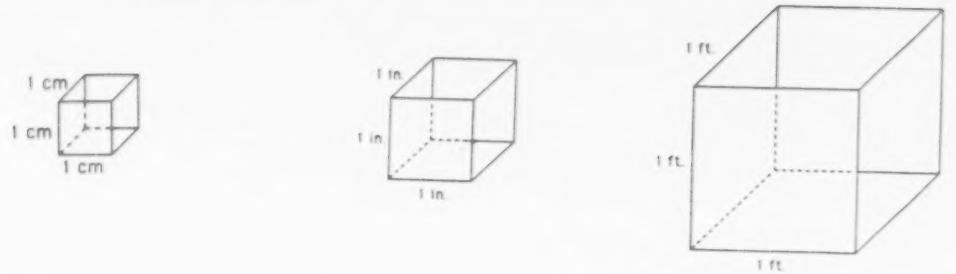
Solution 2: The area may be found by computing the perimeter of a base (2 ft. + 3 ft. + 2 ft. + 3 ft.) and multiplying the perimeter by the height (4 ft.) to determine the lateral area. Since the ends (bases) are to be wrapped too, the area of each base must be added to the lateral area.

perimeter of a base (10 ft.) x height (4 ft.) = 40 sq. ft.
area of a base (6 ft.) x (2) = + 12 sq. ft.
total surface area 52 sq. ft.

Therefore, the 60 sq. ft. of paper for $6.99 would be the amount needed to buy.

ACTIVITY 3 - OUTCOME 1 (Cont'd)

Example 2: Ask students to draw cubes such as the following:



Point out that the volume measures shown previously are "units cubed" (denoted by an exponent 3).

Ask students to measure the length, width and height of the classroom. Have the students figure the volume of the room in cubic feet, cubic inches, cubic meters, and cubic centimeters.

ACTIVITY 4 - OUTCOME 1

Various sized cans are needed for this activity. Tissue boxes, paper towel holders, or containers for canned biscuits may be used.

1. Discuss the lateral area of the different cylinders. By carefully removing the labels, the students can "see" the lateral area formula.
2. Compare the volumes of different sized cans. Students will need to measure height and diameter. Ask if any two cylinders are similar? Why or why not?

ACTIVITY 5 - OUTCOME 1

A rock and a can or a clear glass cylindrical cup are needed for this Activity. Have the students use the cylinder (can) to determine the volume of the rock. Give students time to come up with possibilities.

Measure the diameter of the cylinder. Place a small amount of water in the cylinder and measure the height of the water level. Place the rock in the cylinder and measure the height of the water level. The volume of the water displaced by the rock is the volume of the rock.

AREA: LOCUS

OUTCOME: 1. Describe ellipse, parabola, circle, and hyperbola as the locus of points in a plane.

NOTES FOR THE TEACHER

Prerequisite for Students

Should have some idea as to the appearance of a parabola, hyberbola, circle, and ellipse.

Steps to Success

Good listening skills are crucial to successful activities in which the directions are given orally. Be sure that every student is following the directions accurately before moving to the next step.

ACTIVITY 1 - OUTCOME 1

Take a piece of wax paper. Fold to make a straight line. Mark a point not on the line. With the paper opened up, slide the point along the line creasing the wax paper at each new location of the point on the line. There should be at least 30 folds. Open the paper and observe the figure formed. It should be a parabola. Note that the line is the directrix, and the point is the focus. Have students use this information to write the definition of a parabola as a locus of points in the plane.

AREA: LOCUS

OUTCOMES: 3. Describe conic sections as the intersection of a plane and a cone.

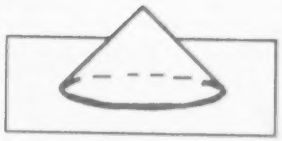
NOTES FOR THE TEACHER

Students need cone-shaped paper cups or cones made from rolled pieces of paper, scissors, pencil, and paper.

ACTIVITY 1 - OUTCOME 3

Direct students to:

1. Place cups or cones on paper with the open side down. Trace around edge. The result is a circle.



2. Press cone flat and clip off one corner of top edge. Open cone by gently pressing creases. Trace around the cone again but do not attempt to trace where the cone lifts from the paper.The result is a parabola. (Note: if two opposite tracings are made, a hyperbola results.)

3. Flatten cone again and clip all the way across at a slant. Place this clipped edge on paper and trace. The result is an ellipse.

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AREA: LOCUS

OUTCOME: 4. **Given specific conditions, describe the locus of points in a plane or in space that satisfy these conditions.**

NOTES FOR THE TEACHER

In teaching about the locus of points in a plane, emphasize that the students will be concerned with locating and describing sets of points which satisfy given conditions. Each time the students see the word "locus" they should think and say "the set of points." Also require the students to use the following steps to determine a locus:

1. Start with the given figure(s).
2. Locate several points that satisfy the given conditions.
3. Draw a smooth, dashed curve or line through the points.
4. Describe the set of points (locus) completely by naming the geometric figure formed and by stating the condition(s) satisfied.

Example 1: Describe the set of points in a plane at a given distance D from a given point P.

Step 1: The given figure is point P. The given distance is D.

Step 2: Locate the points which are D distance from P and in one plane.
Step 3: Describe the set of points. The set of points in a plane at a given distance D from point P is a circle with the center P and the radius with a length of D.

Each time students describe points in a plane which satisfy a given condition or given conditions, have them to describe the set of <u>all</u> points. Use hands, fingers, cardboard, chalk on a string, and many other common objects to help students visualize the figure(s).

NOTES FOR THE TEACHER (Cont'd)

Example 2: Describe the set of points at a given distance D from a given point P.

Step 1: Given:

P D

• |

Step 2: Place a point P on the chalkboard. Use chalk on the end of a piece of string to show points which are D distance from P.

Step 3: Sketch the curve.

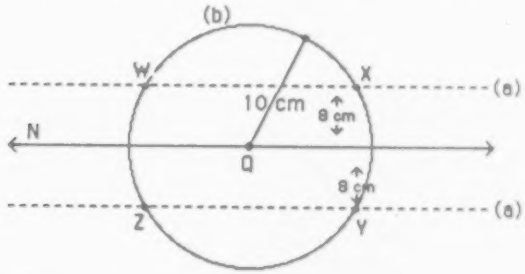
Step 4: Describe the set of points. The set of all points at a given distance D from a given point P is a sphere with center P and radius of length D.

Example 3: Describe the set of points in a plane 8 cm from line N and 10 cm from point Q on line N.

Step 1:

Step 2: Locate points.

a. What are points 8 cm from line N?
b. What are points 10 cm from point Q?



Step 3: The figure is sketched above.

Step 4: Describe the set of points. The set of points in a plane 8 cm from line N and 10 cm from point Q on line N is a set of four points W, X, Y, and Z.

ACTIVITY 1 - OUTCOME 4

Ask the following questions about Example 3:

1. If a line and a point on it are given, could the set of points r distance from the line and d distance from the point be exactly two points? Exactly three points? An infinite number of points? (Give the specific conditions which will yield specified number of points.)
2. Is it possible to have no points of intersection? Explain.
3. What is the set of all points which are 8 cm from line N and 10 cm from point Q on on line N? (If the students have difficulty visualizing the intersection, use a donut cutter to represent the cylinder, and a tomato, potato, or orange to represent the sphere. Actually cut the fruit/vegetable with the cutter, and have the students describe the points.

AREA: LOCUS

OUTCOME: 5. Describe points of concurrency in a triangle as a locus of points: altitudes, medians, angles, bisectors, centroid.

NOTES FOR THE TEACHER

Review the definitions of perpendicular bisector, angle bisector, altitude, and median. Define concurrency.

An overhead projector, felt pen, and wax paper are needed. For best results, draw and pre-fold a few triangles for Activity 1.

ACTIVITY 1 - OUTCOME 5

1. Draw a large acute triangle on the wax paper with a felt pen. Label the vertices. Fold the wax paper gently to touch two vertices together and crease the paper. Touch two more together and crease, then do likewise with the the last two. The creases show up clearly on the overhead. Point out that each crease represents the perpendicular bisector of a side. Where do these lines intersect?

2. Repeat the process with a right triangle, and then with an obtuse triangle. Have the students draw conclusions about where the points of concurrency occur in each triangle.

3. Use the same process to show angle bisectors by folding the wax paper to align the sides of the triangles and creases.

4. The same process can be used to show concurrency of altitudes and medians.

AREA: LOCUS

OUTCOMES:
1. Describe ellipse, parabola, circle, and hyperbola as the locus of points in a plane.
2. Determine intersections of loci.
3. Describe conic sections as the intersection of a plane and a cone.
4. Given specific conditions, describe the locus of points in a plane or in space that satisfy these conditions.
5. Describe points of concurrency in a triangle as a locus of points: altitudes, medians, angle bisectors, and centroid.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Can identify the parts of a triangle: altitude, median, angle bisector, and perpendicular bisector of a line segment.
2. Can classify triangles.

Steps to Success

1. A roll of wax paper and an overhead projector are needed.
2. The teacher should make a cone by plugging the hole in a cone-shaped kitchen funnel. Next, melt enough paraffin to fill the funnel to the desired depth. Let the paraffin set overnight or place in the refrigerator to harden. Remove the paraffin cone from the mold. Be sure to have a knife available for use in the classroom.

ACTIVITY 1 - OUTCOMES 1, 2, AND 3

Use the paraffin cone and a knife to illustrate the intersection of a cone and a plane.

1. Determine the intersection if the plane (knife) intersects the cone so that the plane is perpendicular to the axis of the cone. Illustrate by cutting near the top of the cone. Have the students describe the points. Emphasize the locus of points.
2. Determine the intersection if the plane (knife) intersects the cone so that the plane is perpendicular to the base of the cone or is parallel to the axis. Demonstrate.
3. Determine the intersection if the plane intersects so that it is oblique to the axis of the cone. Demonstrate.

After the students determine the intersection under each condition, lead them in a discussion of the conditions which must be satisfied to generate the set of points for a circle, a parabola, an ellipse, and a hyperbola.

ACTIVITY 2 - OUTCOMES 1, 2, and 3

Have the students find ways in which the parabola is used; for example, a water fountain, the curve of a baseball when it is thrown, or in solid figures called paraboloids; such as, disk receivers for televisions or headlights of a car.

ACTIVITY 3 - OUTCOME 5

Give each student a twelve-inch portion of wax paper. Have the students fold the paper to form a scalene acute triangle. (The teacher should also fold a piece of wax paper and have it ready when the students complete their model. The teacher should place his/her model on the overhead. The lines will show.) Ask the students to fold the paper to form the altitude from each vertex. To do this, they will start the fold through the vertex, and slide the two parts of the opposite side until they coincide. Make a crease. Repeat the process at each vertex. If the students fold carefully, the three altitudes will have exactly one point in common. (The teacher should emphasize that the point of concurrency of the altitudes is the orthocenter of a triangle. Show the point on the overhead.)

ACTIVITY 4 - OUTCOMES 2, 4, and 5

Use the triangles from Activity 3. Have the students fold through a vertex to get an angle bisector. To get the angle bisector, they will need to fold through the vertex, and slide until the two rays of the angle coincide. Make a crease. Repeat the process for each angle. All three will intersect in exactly one point. (The teacher should emphasize that the point of concurrency of the angle bisectors is the incenter of a triangle. Show how a circle can be inscribed in the triangle.)

ACTIVITY 4 - OUTCOMES 2, 4, and 5 (Cont'd)

Ask questions such as the following:

1. What is the set of points in a plane equidistant from the rays of an angle?
2. What is the set of all points equidistant from the rays of an angle?
3. What is the set of all points equidistant from the three sides of a triangle?
4. What is the set of all points equidistant from two adjacent walls in the classroom?
5. What is the set of all points equidistant from the four walls in the classroom?

ACTIVITY 5 - OUTCOMES 2, 4, and 5

Give each student a twelve-inch portion of wax paper. Ask the students to fold the paper to make a scalene triangle. The students should find the perpendicular bisector of each side of the triangle. They should do this by folding the paper so that one end point of the line segment will coincide with the other end point. The students will see that all three perpendicular bisectors intersect in one point. The point is known as the circumcenter of the triangle. Show how a circle can be circumscribed about the triangle.

The teacher should ask questions such as the following:

1. What is the set of points equidistant from any two given points in a plane?
2. What is the set of all points equidistant from two given points? (The perpendicular bisecting plane.)
3. What is the set of points equidistant from three non-collinear points in a plane?
4. What is the set of all points equidistant from three non-collinear points? (A line perpendicular to the plane of the triangle at the circumcenter.)
5. What is the set of points equidistant from three collinear points?

ACTIVITY 6 - OUTCOME 5

Use the triangles from Activity 5. Have students use a midpoint of a side and a vertex to get a median. Fold to get all three medians. The medians intersect in one point, which is known as the centroid of a triangle. Have the students find information on centroid. What is the relationship to the center of gravity? The students can also show by folding that the medians meet in a point that is the point of bisection of each one. Take one median and fold from the midpoint of the side to the point of intersection. Fold congruent segments on the median.

ACTIVITY 7 - OUTCOME 5

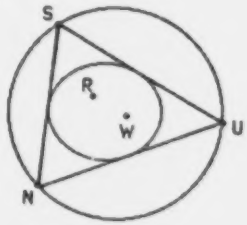
The paper folding described in Activities 3-6, can be done with an obtuse triangle, an equilateral triangle, a right triangle, and/or an isosceles triangle.

Ask what would happen to the centroid, orthocenter, incenter, and circumcenter in each of the above triangles. The class may be divided into groups and let each group work with a different kind of triangle. Emphasize the conditions which are satisfied in determining the points.

ACTIVITY 8 - OUTCOMES 2, 4, and 5

Prepare a transparency which shows the circumcenter and the incenter of a triangle SUN.

Given: R is the incenter of $\triangle$ SUN.
W is the circumcenter of $\triangle$ SUN.



Which point satisfies each of the conditions?

1. It is equidistant from S and N.
2. It is equidistant from rays SU and SN.
3. It is on the perpendicular bisector of $\overline{NU}$.
4. It is on the bisector of angle SUN.
5. What would happen to R and W if $\triangle$ SUN is an equilateral triangle?

AREA: COORDINATE GEOMETRY

OUTCOMES:
1. Understand the special relationships of the number lines that determine the Cartesian plane.
2. Locate points in the coordinate plane.
3. Learn the definitions of the terms "abscissa" and "ordinate."
4. Relate the points in the Cartesian plane to the terms "abscissa" and "ordinate."
5. Use the Pythagorean Theorem to develop the distance formula.
6. Learn the definition of slope.
7. Use the slope formula.
9. Use the point-slope formula to write the equation of a straight line.
10. Given two points, write the equation of the line determined by the two points.
11. State the relationship of the slope of two lines when the lines are parallel and/or the lines are perpendicular.
12. Develop proofs using coordinate methods.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Competency with operations of signed numbers.

Steps to Success

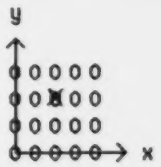
For Activities 1, 2, and 4, prepare the room so that it can be used as the first quadrant of the Cartesian plane. Arrange the desks in rows.

The first vertical row is the positive y-axis.
The first horizontal row is the positive x-axis.

ACTIVITY 1 - OUTCOMES 1, 2, 3, and 4

Let each student determine his/her coordinates.

For example:



The student marked with an X would have coordinates of (2,3).

Ask students to stand if their coordinates satisfy statements such as the following:

a. You have an abscissa of 4.
b. Your abscissa and ordinate are equal.
c. You are a point on the x-axis.
d. You are a point on the y-axis.
e. Your ordinate is 2.
f. Your coordinates are on the line $y = 4$.
g. Your ordinate is twice your abscissa.
h. Your abscissa is half of your ordinate.
i. Your coordinates satisfy the equation: $2x + y = 4$.

As other statements are made, keep in mind that there is a limited number of points in the classroom.

ACTIVITY 2 - OUTCOMES 5, 10, 11, and 12

Material: ball of string

1. Use the room as the first quadrant. Ask the student whose coordinates are (4,3) to stand. Then ask the student whose coordinates are (0,0) to stand. Ask the class to give specific directions of how one can move in the classroom from (0,0) to (4,3). (Students cannot walk across desks to move from one place to another.) Students will develop the idea of moving from (0,0) four spaces on the horizontal line and then three spaces from that point on the vertical line. The teacher then defines the slope as the ratio of the change in y-distance and the change in the x-distance.

ACTIVITY 2 - OUTCOMES 5, 10, 11, and 12 (Cont'd)

2. Use the string and give an end to one student and the ball of string to another student. With the string held taunt, ask the students to find the slope of the line. Do this several times.

3. The teacher should then write the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ on the board.

4. Follow steps in 1 above. In the right triangle where the hypotenuse has end points (0,0) and (4,3). What is the length of the hypotenuse?

   Work several problems similar to this and then derive the distance formula.

ACTIVITY 3 - OUTCOMES 6, 7, and 9

1. Emphasize that if points are collinear, then the slopes of any two line segments of the line are equal. With this emphasis, it is not necessary for students to memorize the point-slope formula and the two point formulas for writing the equation of a line.

2. To find the equation of the line having a slope of 2 and passing through the point (2,-1), let the point (x,y) be another point on the line. By using the slope formula, the slope of the segment between (2,-1) and (x,y) is $\frac{y + 1}{x - 2}$. The slope of the line is 2.

   Since the slopes of the line segments of a line are equal, $\frac{y + 1}{x - 2} = 2$.

   Simplify and write in the standard form of a line.

$$y + 1 = 2(x - 2)$$
$$y + 1 = 2x - 4$$
$$y - 2x = -5 \quad \text{or}$$
$$2x - y = 5$$

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ACTIVITY 3 - OUTCOMES 6, 7, and 9 (Cont'd)

3. To find the equation of the line through the points (4,1) and (-2,3), let (x,y) be another point on the line.

   The slope between (4,1) and (-2,3) is $\frac{3 - 1}{-2 - 4} = \frac{2}{-6} = -\frac{1}{3}$.

   The slope between (4,1) and (x,y) is $\frac{y - 1}{x - 4}$.

   The two slopes are of the same line and must be equal, therefore $\frac{y - 1}{x - 4} = -\frac{1}{3}$.

   Simplify and write in standard form:

   $3(y - 1) = -(x - 4)$

   $3y - 3 = -x + 4$

   $3y + x = 7$

   $3y + x = 7$ or $x + 3y = 7$ is the equation of the line through points (4,1) and (-2,3).

ACTIVITY 4 - OUTCOMES 6, 8, 10, and 11

Use the room as the first quadrant of the Cartesian plane. Ask the students whose coordinates satisfy $x + y = 2$ to stand and remain standing. Ask the students whose coordinates satisfy the $x + y = 3$ to stand. Quiz the students on the characteristics of the two lines. (a) What is the slope of each line? (b) What is the relationship of the two lines? Now ask the students whose coordinates satisfy $x + y = 4$ to stand. Compare this line with $x + y = 2$ and $x + y = 3$. Compare lines such as $x = 4$ and $x = 6$ or $y = 2$ and $y = 3$. Each time students should realize that parallel lines have equal slopes.

Follow the same procedures for lines which are perpendicular.

Compare:	$x + y = 1$	and	$x - y = 1$.
Compare:	$2x + y = 3$	and	$2x - y = 1$.
Compare:	$y = 2$	and	$x = 3$.

ACTIVITY 4 - OUTCOMES 6, 8, 10, and 11 (Cont'd)

Ask the students whose coordinates satisfy $x - 2y = 1$ to stand. Then have the students find at least one line which is parallel to $x - 2y = 1$. Let them decide which students should stand. Now have them write the equation of that line and derive the equation in more than one way.

Follow the same procedure to determine a line perpendicular to $x - 2y = 1$.

NOTE: Students may choose to use the slope formula to derive their equations, or they may decide to use the point-slope formula or y-intercept formula.

AREA: TRANSFORMATIONS

OUTCOMES:
1. Understand basic mapping concepts.
2. Express a function as a mapping.
3. Use informal demonstrations to identify reflections, translations, and rotations.
4. Apply point, line, and plane symmetry to the location of points in a plane or in space.

NOTES FOR THE TEACHER

Prerequisite for Students

Familiarity with the Cartesian plane.

Steps to Success

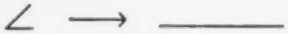
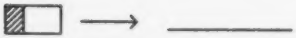
1. If students have a thorough understanding of transformations, their insight to graphing will be greatly enhanced. After teaching the principles of transformations, take several functions and show how changes in x and y affect the graph of the function.
2. The word "transformation" implies that an object is being changed in some way. In a geometric transformation, there are three things to consider: the original figure, a rule that describes the change, and the figure that results after the change.
3. In a plane, a reflection over line "l" is a transformation that maps each point P of the plane onto the point P′ as follows:
   a. If P is on l, P′ = P.
   b. If P is not on l, then l is the perpendicular bisector of PP′.

ACTIVITY 1 - OUTCOMES 1, 3, and 4

Write the rule that describes these transformations.

Answer

Apply that rule to the following figures:

1. 
2.
3.
4.
5. 
6.

Rule: The three sets of figures in the first line are all symmetrical to a vertical line.

1.
2.

ACTIVITY 2 - OUTCOMES 1, 2, 3, and 4

Define a function in two ways and then demonstrate the transformations.

Functions:

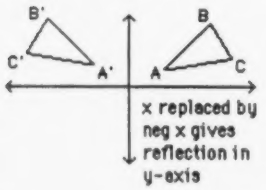
Definition I: Ordered pairs no two of which have the same first element.

Definition II: Mappings

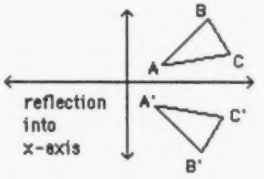
$1 \longrightarrow 2$	$1 \longrightarrow 1$	$1 \longrightarrow 3$	$1 \longrightarrow 4$
$2 \longrightarrow 4$	$2 \longrightarrow 4$	$2 \longrightarrow 5$	$2 \longrightarrow 6$
$3 \longrightarrow 6$	$3 \longrightarrow 9$	$3 \longrightarrow 7$	$3 \longrightarrow 8$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n \longrightarrow 2n$	$n \longrightarrow n^2$	$n \longrightarrow 2n+1$	$n \longrightarrow 2(n+1)$

$(n+1)^2 \longrightarrow n^2 + 2n + 1$

$(x, y) \longrightarrow (-x, y)$

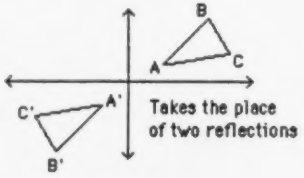


$(x, y) \longrightarrow (x, -y)$



$(x, y) \longrightarrow (-x, -y)$

Reflected into origin



consider

$(x, y) \longrightarrow (x + 3, y)$

$(x, y) \longrightarrow (x, y + 2)$

$(x, y) \longrightarrow (x + k, y + k)$

↓

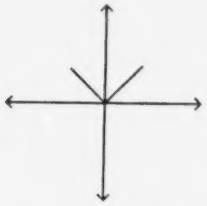
Single transformation which does the job of two.

$(x, y) \longrightarrow (2x, y)$

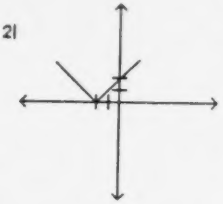
Stretching or compressing

Variations on the Absolute Value Function

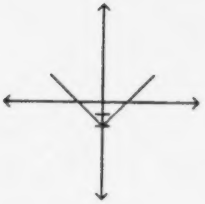
$y = |x|$



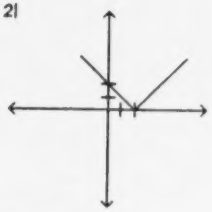
$y = |x + 2|$



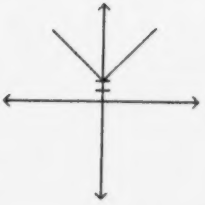
$y = |x| - 2$



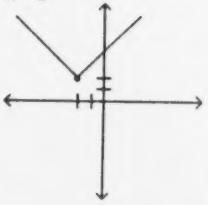
$y = |x - 2|$



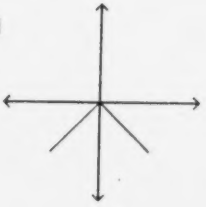
$y = |x| + 2$



$y = |x + 2| + 2$



$y = -|x|$



MATHEMATICS: UNIFIED GEOMETRY AREA: Introduction to Concepts of Geometry

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Geometries:				
		Euclidean	1. Understand the term "Euclidean geometry" and know the fundamental proposition of Euclidean geometry.			
		Non-Euclidean	2. Know that other geometries exist and be able to discuss some general properties of these non-Euclidean geometries.			
*		Key Mathematical Ideas:				
		Undefined terms	3. List, discuss, and represent the undefinded terms of Euclidean geometry.			
		Basic definitions	4. Recognize the definitions of all defined terms of Euclidean geometry.			
*		Sets of Logic:	5. Apply the principles of symbolic logic to the concept of proof.			
		Language of sets				
		Principles of logic				
*		Induction, a method of discovery	6. Distinguish between inductive and deductive reasoning.			
*		Deduction, a method of proof				

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Introduction to a formal geometric proof	7. Use deductive reasoning to construct a formal geometric proof.			
*		Algebraic symbols and language	8. Demonstrate the ability to solve algebraic equations.			
*		Tools of construction				
*		Basic constructions: parallel lines, perpendicular lines, and equal line segments	9. Use a compass and straight edge to make geometric constructions including parallel lines, perpendicular lines, and equal line segments.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Introduction to Points, Lines, and Planes

C OF S	S T A N	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
*		Intuitive ideas and undefined terms	Included in outcomes for Area: Introduction to Concepts of Geometry.			
*		Basic definitions				
*		Postulates and theorems				
*		Symbols and notations Algebraic Geometric				
*		Expressed relations In coordinate systems Using sets and subsets				
*		Line segments and rays				

MATHEMATICS: UNIFIED GEOMETRY AREA: Introduction to Angles

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Measurement of angles	1. Apply the protractor theorem to the measurement of angles and use measurement to determine congruence.			
*		Classification of angles	2. Classify angles by types, according to measurement.			
*		Special relation between pairs of angles: adjacent angles, vertical angles, complementary, and supplementary	3. Describe special relationships between pairs of angles: i.e., (1) adjacent angles, (2) vertical angles, (3) complementary angles, (4) supplementary angles, (5) corresponding angles (reference parallel lines) (6) alternate interior angles of parallel lines.			
*		Congruence of angles				
*		Initial proofs relating to angles:				
		Differences between direct and indirect proof				
		Application of elementary logic to proofs				

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Parallel lines and planes				
		Construction showing relationship				
		Identification of angles formed by transversal of parallel lines				
		Proof of lines being parallel	1. Develop deductive proofs concerning parallel lines.			
		Parallelism in non-Euclidean geometry				
*		Perpendicular lines and planes				
		Construction of perpendicular lines				
		Proof of lines being perpendicular	2. Develop deductive proofs concerning the perpendicularity of lines and planes.			
		Application of theorems				
		Perpendicular lines in a plane				

MATHEMATICS: UNIFIED GEOMETRY AREA: Triangles

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Classification of triangles				
		According to sides	1. Classify triangles according to sides.			
		According to angles	2. Classify triangles according to angles.			
*		Congruence of triangles				
		Correspondence of parts of triangles				
		Postulates of congruence	3. Use the postulates of congruence to prove triangles congruent.			
*		Original proofs and exercises	4. Prove segments and angles congruent by proving that they are corresponding parts of congruent triangles.			
*		Similarity of triangles				
		Similarity and proportionality of sides of triangles	5. Prove triangles similar.			
		Congruence and similarity	6. Use the corresponding sides of similar triangles to set up proportions.			
		Equivalence relationship	7. Solve proportions.			

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Application of congruency				
		Sum of angles of a triangle	8. Prove that the sum of the angles of a triangles equals 180 degrees.			
			9. Solve problems using the sum of the angles theorem and its corrolaries.			
		Construction of congruent triangles				
*		The Pythagorean Theorem				
		Algebraic proof	10. Know the Pythagoream Theorem.			
		Converse of Pythagorean Theorem	11. Apply the Pythagorean Theorem to the solution of problems.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Application of Congruent Triangles

COFS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Quadrilaterals: parallelograms, rectangles, rhombuses, and trapezoids Other polygons	1. Apply the theorems of congruent triangles to the proof of theorems concerning parallelograms, rectangles, rhombuses, trapezoids, and other polygons.			
*		Inequalities in triangles	2. Prove triangle inequality theorems. 3. Apply triangle inequality theorems to the solution of problems.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Similar Polygons

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Definitions and properties	1. Know the definitions and properties of similar polygons.			
*		Ratio and proportion	2. Apply the definitions and properties of similar polygons to the proof of other theorems and the solution of problems.			
*		Special right triangles	3. Know the right triangle definitions of the sine, cosine, and tangent functions.			
			4. Use the sine, cosine, and tangent functions to solve problems.			
		30-60-90 degree triangles	5. Apply the properties of the 30-60-90 degree triangle to determining the values of the sine, cosine and tangent of a 30-degree angle and a 60-degree angle.			
		45-45-90 degree triangles	6. Apply the properties of the 45-45-90 degree triangle to determining the values of the sine, cosine, and tangent of a 45-degree angle.			
*		Trigonometry				
		Trigonometric relations in a right triangle	Included in special right triangles outcomes.			
		Law of sines and cosines				
		Application to solution of practical problems				

MATHEMATICS: UNIFIED GEOMETRY — AREA: Similar Polygons (continued)

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Areas of Polygons				
		Area of polygonal regions	7. Develop formulas for finding the area of polygonal regions.			
		Formulas for areas of polygons	8. Use formulas to find the area of polygons.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Circles

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Line segments related to circles: tangents, chords, secants, inscribed polygons, and circumscribed polygons	1. Identify and define: tangents, chords, secants, inscribed polygons, circumscribed polygons, central angles, inscribed angles, angles formed by a tangent and a chord, angles formed by two secants intersecting inside the circle, and by two secants intersecting outside the circle.			
*		Angles and line segments: central angles and inscribed angles	2. Determine the measure of central angles, inscribed angles, angles formed by a tangent and a chord, angles formed by two secants intersecting inside the circle, and angles formed by two secants intersecting outside the circle.			
			3. Understand and work problems with: concentric circles, intersecting circles, disjunct circles, internally tangent circles, and externally circles.			
*		Measurement of circles: circumference and area	4. Find the circumference of a circle.			
			5. Find the area of a circle.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Circles (Cont'd)

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Conic sections: ellipses, parabolas, circles, and hyperbolas	Included in outcomes for Area: Locus.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Geometric Solids

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Individual properties: cylinders, polyhedrons, prisms and parallelopipeds, cones and pyramids, and spheres				
*		Formulas: area and volume Application to practical problems	1. Identify and use formulas to find the area and volume of: cylinders, rectangular solids, pyramids, cones, and spheres.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Locus

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Introduction				
		Conditions determining a figure				
		Description of a locus	1. Describe ellipse, parabola, circle, and hyperbola as the locus of points in a plane.			
*		Basic locus theorem				
*		Intersections of loci	2. Determine intersections of loci.			
*		Locus in three dimensions	3. Describe conic sections as the intersection of a plane and a cone.			
			4. Given specific conditions, describe the locus of points in a plane or in space that satisfy these conditions.			
*		Points of concurrency in a triangle: altitudes, medians, angle bisectors, and centroid	5. Describe points of concurrency in a triangle as a locus of points: altitudes, medians, angle bisectors, and centroid.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Coordinate Geometry

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		The coordinate plane	1. Understand the special relationships of the number lines that determine the Cartesian plane.			
			2. Locate points in the coordinate plane.			
			3. Learn the definitions of the terms "abscissa" and "ordinate."			
			4. Relate the points in the Cartesian plane to the terms "abscissa" and "ordinate."			
*		The distance formula	5. Use the Pythagorean Theorem to develop the distance formula.			
*		Representation of lines and planes in coordinate planes by algebraic equations	6. Learn the definition of slope.			
			7. Use the slope formula.			
			8. Use the midpoint formula.			
			9. Use the point-slope formula to write the equation of a straight line.			
			10. Given two points, write the equation of the line determined by the two points.			
			11. State the relationship of the slopes of two lines when: the lines are parallel and/or the lines are perpendicular.			
*		Proofs using coordinate methods	12. Develop proofs using coordinate methods.			

MATHEMATICS: UNIFIED GEOMETRY AREA: Transformations

COS	STAN	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
*		Basic mapping: congruence mapping and isometry transformations	1. Understand basic mapping concepts. 2. Express a function as a mapping.			
*		Composite mapping: rotations, translations, dilations, and reflections	3. Use informal demonstrations to identify reflections, translations, and rotations. 4. Apply point, line, and plane symmetry to the location of points in a plane or in space.			

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